

## Bronze Outcome 1 - Exponential equations

**RIGOUR**

Solving Exponential Equations

Outcome 1

Exponential Equations

Solve the following exponential equations...

1.  $3^x = 7.9$     2.  $9^x = 1.2$

3.  $5^x = 0.8$     4.  $2^x = 0.3$

5.  $e^x = 6.5$     6.  $e^{2x} = 4.7$

7.  $e^{5x} = 0.32$     8.  $e^{8x} = 0.1$

9.  $e^{-3x} = 3.9$     10.  $e^{-x} = 2.6$

Examples...

Take 'ln' of both sides  
Bring the x down  
Cancel out the 'ln'.  
(since  $\ln e = \log_e e = 1$ )

Solve the following equations for x...

Divide

$7^x = 2.4$      $e^{2x} = 3.8$

$\ln 7^x = \ln 2.4$      $\ln e^{2x} = \ln 3.8$

$x \ln 7 = \ln 2.4$      $2x \ln e = \ln 3.8$

$x = \frac{\ln 2.4}{\ln 7}$      $x = \frac{\ln 3.8}{2}$

$= 0.4499$      $= 0.668$

1. $x = 1.881$	2. $x = 0.083$
3. $x = -0.139$	4. $x = -1.737$
5. $x = 1.872$	6. $x = 0.774$
7. $x = -0.228$	8. $x = -0.288$
9. $x = -0.454$	10. $x = -0.956$

①  $3^x = 7.9$   
 $\ln 3^x = \ln 7.9$   
 $x \ln 3 = \ln 7.9$   
 $x = \frac{\ln 7.9}{\ln 3}$   
 $= 1.881$

②  $9^x = 1.2$   
 $\ln 9^x = \ln 1.2$   
 $x \ln 9 = \ln 1.2$   
 $x = \frac{\ln 1.2}{\ln 9}$   
 $= 0.083$

③  $5^x = 0.8$   
 $\ln 5^x = \ln 0.8$   
 $x \ln 5 = \ln 0.8$   
 $x = \frac{\ln 0.8}{\ln 5} = -0.139$

④  $2^x = 0.3$   
 $\ln 2^x = \ln 0.3$   
 $x \ln 2 = \ln 0.3$   
 $x = \frac{\ln 0.3}{\ln 2} = -1.737$

⑤  $e^{5x} = 0.32$   
 $\ln e^{5x} = \ln 0.32$   
 $5x \ln e = \ln 0.32$   
 $5x = \frac{\ln 0.32}{1}$   
 $x = \frac{\ln 0.32}{5} = -0.288$

⑥  $e^{8x} = 0.1$   
 $\ln e^{8x} = \ln 0.1$   
 $8x \ln e = \ln 0.1$   
 $8x = \frac{\ln 0.1}{1}$   
 $x = \frac{\ln 0.1}{8} = -0.288$

⑤  $e^x = 6.5$   
 $\ln e^x = \ln 6.5$   
 $x \ln e = \ln 6.5$   
 $x = \frac{\ln 6.5}{1}$   
 $= 1.872$

⑥  $e^{2x} = 4.7$   
 $\ln e^{2x} = \ln 4.7$   
 $2x \ln e = \ln 4.7$   
 $2x = \frac{\ln 4.7}{1}$   
 $x = \frac{\ln 4.7}{2} = 0.774$

⑦  $e^{-3x} = 3.9$   
 $\ln e^{-3x} = \ln 3.9$   
 $-3x \ln e = \ln 3.9$   
 $-3x = \frac{\ln 3.9}{1}$   
 $x = \frac{\ln 3.9}{-3} = -0.454$

⑩  $e^{-x} = 2.6$   
 $\ln e^{-x} = \ln 2.6$   
 $-x \ln e = \ln 2.6$   
 $-x = \frac{\ln 2.6}{1}$   
 $x = \frac{\ln 2.6}{-1} = -0.956$

## Silver Outcome 2 - Evaluating exponential equations

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Solving Exponential Equations

Outcome 2

Evaluating Exponential Equations

Evaluate the following exponential equations...

1.  $P_t = P_0 e^{-0.009t}$   
Evaluate  $P_t$  when  $P_0 = 800$  and  $t = 700$ .

2.  $M_t = M_0 e^{-0.023t}$   
Evaluate  $M_t$  when  $M_0 = 700$  and  $t = 40$ .

3.  $I_t = I_0 e^{-0.006t}$   
Evaluate  $I_t$  when  $I_0 = 97$  and  $t = 25$ .

4.  $N_t = N_0 e^{-0.064t}$   
Evaluate  $N_t$  when  $N_0 = 120$  and  $t = 200$ .

Examples...

Sub into formula

$A_t = A_0 e^{-0.003t}$   
Evaluate  $A_t$  when  $A_0 = 400$  and  $t = 500$ .  
 $A_t = 400 \times e^{-0.003 \times 500}$   
 $= 400e^{-1.5} = 89.25$

$V_t = V_0 e^{-0.0157t}$   
Evaluate  $V_t$  when  $V_0 = 28$  and  $t = 600$ .  
 $28 = V_0 \times e^{-0.0157 \times 600}$   
 $28 = V_0 e^{-9.42}$      $V_0 = 28 \div e^{-9.42}$   
 $= 345.312 \cdot 30$

1. 1.469	2. 278.96
3. 112.70	4. 43 466 093.95

①  $P_t = 800 \times e^{-0.009 \times 700}$   
 $= 800 \times e^{-6.3} = 1.469$

②  $M_t = 700 \times e^{-0.023 \times 40}$   
 $= 700 \times e^{-0.92} = 278.96$

③  $97 = I_0 e^{-0.006 \times 25}$   
 $97 = I_0 e^{-0.15}$   
 $I_0 = 97 \div e^{-0.15} = 112.70$

④  $120 = N_0 e^{-0.064 \times 200}$   
 $120 = N_0 e^{-12.8}$      $N_0 = 120 \div e^{-12.8}$   
 $= 43 466 093.95$

## Gold Outcome 3 - Finding the "half life"

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Solving Exponential Equations

**Outcome 3**

**Finding the 'half life'**

$P_t = P_0 e^{-0.009t}$   
Calculate the half life.

$M_t = M_0 e^{-0.023t}$   
Calculate the half life.

$I_t = I_0 e^{-0.006t}$   
How long will it take to double in size?

$N_t = N_0 e^{-0.074t}$   
How long will it take to drop to 67% of its original value?

**Examples...**

Find the half life of the substance.

$e^{-0.003t} = 0.5$

$\ln e^{-0.003t} = \ln 0.5$

$-0.003t \ln e = \ln 0.5$

$t = \frac{\ln 0.5}{-0.003} = 231.0 \text{ years}$

1. 77.02 years    2. 30.14 years  
3. 115.52 years    4. 5.41 years

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①  $e^{-0.009t} = 0.5$  (for half life set = 0.5)

$\ln e^{-0.009t} = \ln 0.5$

$-0.009t \ln e = \ln 0.5$

$t = \frac{\ln 0.5}{-0.009} = 77.02 \text{ years}$

②  $e^{-0.023t} = 0.5$

$\ln e^{-0.023t} = \ln 0.5$

$-0.023t \ln e = \ln 0.5$

$t = \frac{\ln 0.5}{-0.023} = 30.14 \text{ years}$

③  $e^{0.006t} = 2$  (to double set = 2)

$\ln e^{0.006t} = \ln 2$

$0.006t \ln e = \ln 2$

$t = \frac{\ln 2}{0.006} = 115.52 \text{ years}$

④  $e^{-0.074t} = 0.67$  ( $67\% = \frac{67}{100} = 0.67$ )

$\ln e^{-0.074t} = \ln 0.67$

$-0.074t \ln e = \ln 0.67$

$t = \frac{\ln 0.67}{-0.074} = 5.41 \text{ years}$