

## Bronze Outcome 1 - Evaluating expressions

**RIGOUR**

Simplifying a Numerical Expression

**Outcome 1**

Evaluating Expressions

Evaluate the following logarithmic expressions...

$\log_2 32$	$\log_3 27$
$\log_4 64$	$\log_5 25$
$\log_{10} 10\,000$	$\log_6 6$
$\log_9 1$	$\log_{64} 8$
$\log_2 \frac{1}{2}$	$\log_8 2$

**Examples...**

Evaluate the following expressions...

base number

$\log_2 8 = 3$   $\log_4 16 = 2$

$2^3 = 8$   $4^2 = 16$

power

$\log_5 1 = 0$   $\log_9 3 = \frac{1}{2}$

$5^0 = 1$   $9^{\frac{1}{2}} = 3$

1. 5	2. 3
3. 3	4. 2
5. 4	6. 1
7. 0	8. $\frac{1}{2}$
9. -1	10. $\frac{1}{3}$

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- $\log_2 32 = 5$   
(since  $2^5 = 32$ )
- $\log_3 27 = 3$   
(since  $3^3 = 27$ )
- $\log_4 64 = 3$   
(since  $4^3 = 64$ )
- $\log_5 25 = 2$   
(since  $5^2 = 25$ )
- $\log_{10} 10\,000 = 4$   
(since  $10^4 = 10\,000$ )
- $\log_2 \frac{1}{2} = -1$   
(since  $2^{-1} = \frac{1}{2}$ )
- $\log_9 1 = 0$   
(since  $9^0 = 1$ )
- $\log_{64} 8 = \frac{1}{2}$   
(since  $64^{\frac{1}{2}} = \sqrt{64} = 8$ )
- $\log_2 \frac{1}{2} = -1$   
(since  $2^{-1} = \frac{1}{2}$ )
- $\log_8 2 = \frac{1}{3}$   
(since  $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$ )
- $\log_6 6 = 1$   
(since  $6^1 = 6$ )

## Silver Outcome 2 - Combining logs

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Simplifying a Numerical Expression

**Outcome 2**

Combining Logs

Simplify the following logarithmic expressions...

- $\log_4 7x + \log_4 3x$
- $\log_6 4x + \log_6 9x^3$
- $\log_8 15x^9 - \log_8 5x^5$
- $\log_3 8x^6 - \log_3 4x^5$
- $\log_2 2 + \log_2 6 - \log_2 3$
- $\log_6 8 + \log_6 3 - \log_6 4$

**Examples...**

You can only add/subtract logs if they have the SAME base.

\*When adding logs you MULTIPLY the terms together\*

$\log_5 3x + \log_5 2x = \log_5 6x^2$

$\log_7 20x^8 - \log_7 5x^3$

$= \log_7 \left( \frac{20x^8}{5x^3} \right) = \log_7 4x^5$

\*When subtracting you DIVIDE the terms\*

$\log_2 6 + \log_2 8 - \log_2 3$

$= \log_2 48 - \log_2 3$

$= \log_2 16 = 4$

\*Then subtract!\*

A logarithm is EQUAL to the POWER you raise the BASE to get the NUMBER.

1. $\log_4 21x^2$	2. $\log_3 36x^4$
3. $\log_3 3x^4$	4. $\log_2 2x$
5. 2	6. 1

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- $\log_4 7x + \log_4 3x$   
 $= \log_4 21x^2$  (since  $7x \times 3x = 21x^2$ )
- $\log_6 4x + \log_6 9x^3$   
 $= \log_6 36x^4$  (since  $4x \times 9x^3 = 36x^4$ )
- $\log_8 15x^9 - \log_8 5x^5$   
 $= \log_8 3x^4$  (since  $\frac{15x^9}{5x^5} = 3x^4$ )
- $\log_3 8x^6 - \log_3 4x^5$   
 $= \log_3 2x$  (since  $\frac{8x^6}{4x^5} = 2x$ )
- $\log_2 2 + \log_2 6 - \log_2 3$   
 $= \log_2 12 - \log_2 3 = \log_2 4 = 2$
- $\log_6 8 + \log_6 3 - \log_6 4$   
 $= \log_6 24 - \log_6 4 = \log_6 6 = 1$

## Gold Outcome 3 - Bringing stuff "up/down"

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Simplifying a Numerical Expression

**Outcome 3**

"Bringing Stuff Up/Down"

Simplify the following logarithmic expressions...

- $\log_2(x+3) - 4\log_2 3$
- $\log_4(x-6) + 2\log_4 5$
- $5\log_8 2 - \log_8(2x+1)$
- $2\log_8 4 + \log_8 4$
- $4\log_9 3 - 2\log_9 3$
- $2\log_{36} 12 - \log_{36} 24$

**Examples...**

1. Bring stuff up/down  
2. Then combine

$\log_8 4 + 2\log_8 3 = \log_8 4 + \log_8 3^2$   
 $= \log_8 4 + \log_8 9 = \log_8 36 = 2$

$\log_5(x-7) - 3\log_5 2$

$= \log_5(x-7) - \log_5 8$

$= \log_5 \left( \frac{x-7}{8} \right)$

A logarithm is EQUAL to the POWER you raise the BASE to get the NUMBER.

1. $\log_2(x+3)/81$	2. $\log_8 25(x-6)$
3. $\log_8 32/(2x+1)$	4. 2
5. 1	6. $\frac{1}{2}$

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- $\log_2(x+3) - 4\log_2 3$   
 $= \log_2 \left( \frac{x+3}{81} \right)$
- $\log_4(x-6) + 2\log_4 5$   
 $= \log_4 25(x-6)$
- $\log_8 32 - \log_8(2x+1)$   
 $= \log_8 \left( \frac{32}{2x+1} \right)$
- $\log_8 16 + \log_8 4$   
 $= \log_8 64 = 2$  (since  $8^2 = 64$ )
- $\log_9 81 - \log_9 9 = \log_9 9 = 1$  (since  $9^1 = 9$ )

$$\begin{aligned} \log_{36} 144 - \log_{36} 24 \\ &= \log_{36} 6 \quad \left( \frac{144}{24} = 6 \right) \\ &= \frac{1}{2} \quad \left( \text{since } 36^{\frac{1}{2}} = 6 \right) \end{aligned}$$