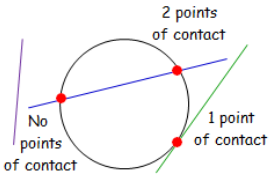
Outcome 1 – 2 points of contact



Bronze example

Examples...

Find the coordinates of the points of intersection of the line $y = 3x + 4$ and the circle $x^2 + y^2 - 10x - 8y - 184 = 0$.

$x^2 + (3x + 4)^2 - 10x - 8(3x + 4) - 184 = 0$

$x^2 + 9x^2 + 24x + 16 - 10x - 24x - 32 - 184 = 0$

$10x^2 - 10x - 200 = 0$ — 1. Sub in wherever there is a 'y'

$10(x^2 - x - 20) = 0$ — 2. Multiply out and gather like terms

$10(x - 5)(x + 4) = 0$ — 3. Factorise

$x = 5 \quad x = -4$ — 4. Solve

$y = 15 + 4 = 19$ — 5. Sub in to get y

$y = -12 + 4 = -8$

$(-4, -8) \quad (5, 19)$ — 6. Write coords

Bronze questions

Calculate the points of intersection between the following circles and straight lines...

1. $x^2 + y^2 + 7x + 4y - 15 = 0$ and $y = 2x - 5$
2. $x^2 + y^2 + 4x - 4y - 41 = 0$ and $y = x - 3$
3. $x^2 + y^2 - 2x + 10y - 72 = 0$ and $y = x - 6$
4. $x^2 + y^2 + 6x + 8y - 235 = 0$ and $x = 9 - 2y$
5. $x^2 + y^2 - 38x - 6y + 30 = 0$ and $4x - y = 5$

Outcome 2 – 1 point of contact

Silver example

Examples... Show that the line $y = x - 1$ is a tangent to the circle $x^2 + y^2 - 8x + 6y + 7 = 0$ and find the coordinates of the point of contact.

$x^2 + (x - 1)^2 - 8x + 6(x - 1) + 7 = 0$

$x^2 + x^2 - 2x + 1 - 8x + 6x - 6 + 7 = 0$

$2x^2 - 4x + 2 = 0$ — 1. Sub in wherever there is a 'y'

$2(x^2 - 2x + 1) = 0$ — 2. Multiply out and gather like terms

$2(x - 1)(x - 1) = 0$ — 3. Factorise

$x = 1$

Since only one point of contact, line is tangent. — 4. Solve

$y = 1 - 1 = 0$ — 5. Statement

$(1, 0)$ — 6. Sub in to get y

7. Write coords

Silver questions

Show that the following lines are tangents to the circles below and find the points of contact...

1. $x^2 + y^2 - 14x - 8y + 55 = 0$ and $y = 3x - 7$
2. $x^2 + y^2 - 122x + 40y + 71 = 0$ and $y = 7x + 3$
3. $(x - 6)^2 + (y + 1)^2 = 152$ and $y = 2x - 15$
4. $x^2 + y^2 + 5x - 10y + 30 = 0$ and $x = 5 - 2y$
5. $x^2 + y^2 - 4x - 6y + 11 = 0$ and $y - x + 1 = 0$

Outcome 3 – No points of contact

Gold example

Examples... Show that the line $y = 2x - 1$ does not intersect the circle with equation $x^2 + y^2 + 2x - y + 8 = 0$.

$x^2 + (2x - 1)^2 + 2x - (2x - 1) + 8 = 0$

$x^2 + 4x^2 - 4x + 1 + 2x - 2x + 1 + 8 = 0$

$5x^2 - 4x + 10 = 0$ — 1. Sub in wherever there is a 'y'

This quadratic does NOT factorise! — 2. Multiply out and gather like terms

$b^2 - 4ac = 16 - 4(50)$

$= 16 - 200 = -184$ — 3. $b^2 - 4ac$

Since $b^2 - 4ac < 0$, there are no real roots, therefore the line does not intersect the circle. — $a = 5$, $b = -4$, $c = 10$

4. THREE part statement!

Gold questions

Show that the following lines do NOT intersect these circles...

1. $x^2 + y^2 + 10x - 10y + 36 = 0$ and $y = x + 4$
2. $x^2 + y^2 - 6x + 4y + 6 = 0$ and $y = 3x - 2$
3. $(x - 6)^2 + (y - 3)^2 = 4$ and $y = 2x + 5$
4. $x^2 + y^2 + 2x - 6y + 9 = 0$ and $y = 2 - x$
5. $x^2 + y^2 - 6x + 2y - 19 = 0$ and $y - x = 4$

Bronze Answers

1. (-1, -7), (2, -1)
2. (-2, -5), (5, 2)
3. (-6, -12), (8, 2)
4. (-11, 10), (13, -2)
5. (1, -1), (5, 15)

Silver Answers

1. (4, 5)
2. (-2, -11)
3. (4, -7)
4. (-3, 4)
5. (3, 2)

Gold Answers

1. $b^2 - 4ac = -32$
2. $b^2 - 4ac = -44$
3. $b^2 - 4ac = -704$
4. $b^2 - 4ac = -8$
5. $b^2 - 4ac = -24$