

## Outcome 1 - Basic Integration

### Bronze examples

**Examples...**

1. Add 1 to the power
2. Divide by new power
3. Add the constant of integration!

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$$

$$\int x^7 dx = \frac{x^8}{8} + c \quad \left(\frac{1}{8}x^8 + c\right)$$

$$\int 20x^3 dx = 5x^4 + c \quad \frac{20x^4}{4}$$

$$\int 8 dx = 8x + c \quad \text{What do I differentiate to get an answer of 8?}$$

$$\int 4x^3 + 8x^2 - 12x + 3 dx = x^4 + \frac{8x^3}{3} - 6x^2 + 3x + c \quad \text{Integrate each term!}$$

### Bronze questions

Calculate the following...

1.  $\int x^7 dx$
2.  $\int x^{15} dx$
3.  $\int 2x^8 dx$
4.  $\int 9x^4 dx$
5.  $\int 3x^2 dx$
6.  $\int 25x^4 dx$
7.  $\int 9x^{-3} dx$
8.  $\int 2x^{-3} dx$
9.  $\int 6x^3 + 8x^2 - 16x + 5 dx$
10.  $\int 16x^3 + 6x^2 - 7x + 11 dx$

## Outcome 2 - Integration with fractions and indices

### Silver examples

**Examples...**

1. Add 1 to the power
2. Divide by new power
3. Add the constant of integration!

$$\int \frac{6}{7}x^3 dx = \int \frac{6x^3}{7} dx = \frac{6x^4}{7 \times 4} = \frac{6x^4}{28} = \frac{3x^4}{14} + c$$

$$\int 3\sqrt{x} dx = \int 3x^{\frac{1}{2}} dx = \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} = 2x^{\frac{3}{2}} + c$$

$$\int \frac{12}{\sqrt[3]{x}} dx = \int 12x^{-\frac{1}{3}} dx = \frac{12x^{\frac{2}{3}}}{\frac{2}{3}} = 18x^{\frac{2}{3}} + c$$

### Silver questions

Calculate the following...

1.  $\int x^{\frac{4}{9}} dx$
2.  $\int \frac{2}{3}x^3 dx$
3.  $\int \frac{1}{5}x^5 dx$
4.  $\int \frac{1}{6}x^{\frac{1}{8}} dx$
5.  $\int \sqrt[10]{x} dx$
6.  $\int 8\sqrt[4]{x} dx$
7.  $\int \sqrt{x^9} dx$
8.  $\int \frac{8}{\sqrt[8]{x}} dx$

## Outcome 3 - Harder Integration!

### Gold examples

**Examples...**

1. Add 1 to the power
2. Divide by new power
3. Add the constant of integration!

$$\int \frac{x+1}{\sqrt{x}} dx = \int \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}} dx$$

**\*\*Split up as separate fractions!\*\***

$$\int \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}} dx = \int x^{\frac{1}{2}} + x^{-\frac{1}{2}} dx$$

$$\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = \frac{2x^{\frac{3}{2}}}{3} + 2x^{\frac{1}{2}} + c$$


$$\frac{3}{2} \cdot \frac{1}{2} = \frac{2\sqrt{x^3}}{3} + 2\sqrt{x} + c$$


### Gold questions


Calculate the following...


1.  $\int \frac{x^9 + x}{x} dx$
2.  $\int \frac{x^4 + 3}{x^2} dx$
3.  $\int \frac{6x + 8}{x^3} dx$
4.  $\int \frac{x - 10}{\sqrt{x}} dx$
5.  $\int \frac{x + 4}{\sqrt[4]{x}} dx$
6.  $\int \frac{x - 7}{\sqrt{x^5}} dx$


## Bronze Answers


  $\frac{x^7}{7} + c$


  $\frac{x^{16}}{16} + c$


  $\frac{9x^5}{5} + c$


  $\frac{11x^8}{8} + c$


  $x^3 + c$

  $5x^5 + c$


  $-\frac{9x^{-2}}{2} + c$


  $-x^{-2} + c$


  $\frac{3x^4}{2} + \frac{8x^3}{3} - 8x^2 + 5x + c$


  $4x^4 + 2x^3 - \frac{7x^2}{2} + 11x + c$


## Silver Answers


  $\frac{9x^{\frac{13}{9}}}{13} + c$


  $\frac{x^4}{6} + c$


  $\frac{x^6}{30} + c$

  $\frac{4x^{\frac{9}{8}}}{27} + c$


  $\frac{10x^{\frac{11}{10}}}{11} + c$


  $\frac{32x^{\frac{5}{4}}}{5} + c$


  $\frac{2x^{\frac{11}{2}}}{11} + c$


  $\frac{64x^{\frac{7}{8}}}{7} + c$


## Gold Answers


  $\frac{x^9}{9} + x + c$

  $\frac{x^3}{3} - \frac{3}{x} + c$

  $-\frac{6}{x} - \frac{4}{x^2} + c$

  $\frac{2\sqrt{x^3}}{3} - 20\sqrt{x} + c$

  $\frac{4\sqrt[4]{x^7}}{7} + \frac{16\sqrt[4]{x^3}}{3} + c$

  $-\frac{2}{\sqrt{x}} + \frac{14}{3\sqrt{x^3}} + c$