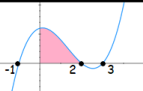


Outcome 1 - Between a curve and the x-axis

Bronze example

Example... The curve $y = x^3 - 4x^2 + x + 6$ intersects the x-axis at points $(-1, 0)$, $(2, 0)$ and $(3, 0)$. Calculate the shaded area.



$$\int_0^2 x^3 - 4x^2 + x + 6 \, dx \quad \int_a^b \text{curve} \, dx$$

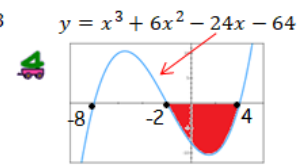
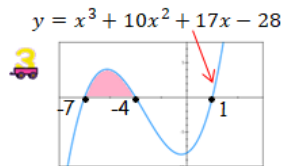
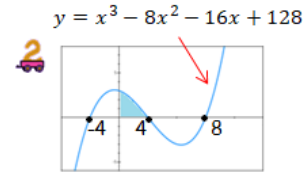
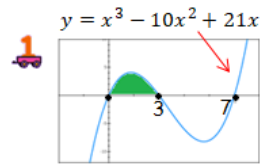
$$= \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + 6x \right]_0^2$$

$$= \left(4 - \frac{32}{3} + 2 + 12 \right) - (0) = \frac{22}{3} \text{ units}^2$$

$$18 - \frac{32}{3} = \frac{54}{3} - \frac{32}{3}$$

Bronze questions

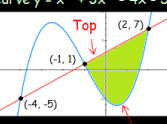
Calculate the following shaded areas between the curves and the x-axis...



Outcome 2 - Between a line and a curve

Silver example

Example... The curve $y = x^3 + 3x^2 - 4x - 5$ intersects the line $y = 2x + 3$ at points $(-4, -5)$, $(-1, 1)$ and $(2, 7)$. Calculate the shaded area.



$$\int_a^b \text{top} - \text{bottom} \, dx$$

$$\int_{-4}^2 2x + 3 - (x^3 + 3x^2 - 4x - 5) \, dx$$

$$= \int_{-4}^2 -x^3 - 3x^2 + 6x + 8 \, dx$$

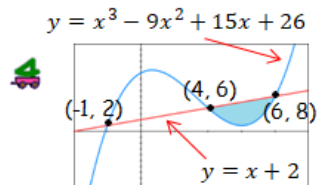
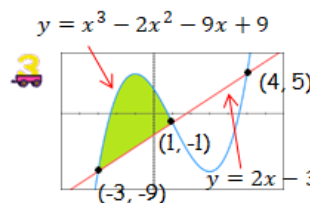
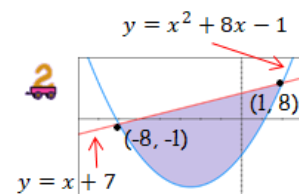
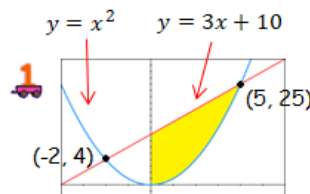
$$= \left[-\frac{x^4}{4} - x^3 + 3x^2 + 8x \right]_{-4}^2$$

$$= \left(-4 - 8 + 12 + 16 \right) - \left(-\frac{1}{4} + 1 + 3 - 8 \right)$$

$$= (16) - \left(-\frac{17}{4} \right) = \frac{81}{4} \text{ units}^2$$

Silver questions

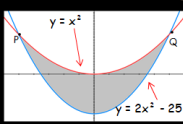
Calculate the following shaded areas between the curves and the lines...



Outcome 3 - Between 2 curves

Gold example

Example The curve $y = 2x^2 - 25$ intersects the curve $y = x^2$ at points P and Q. Calculate the shaded area.



1. Find the limits (set them equal, set = 0, factorise)

$$2x^2 - 25 = x^2$$

$$x^2 - 25 = 0$$

$$(x + 5)(x - 5) = 0 \quad x = -5, x = 5$$

2. $\int_a^b \text{top} - \text{bottom} \, dx$

$$\int_{-5}^5 x^2 - (2x^2 - 25) \, dx$$

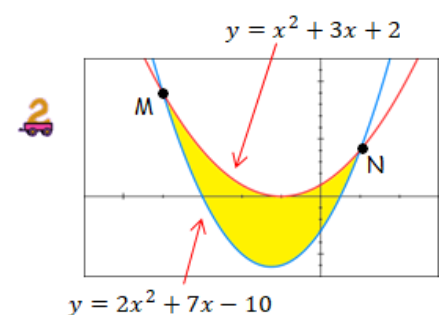
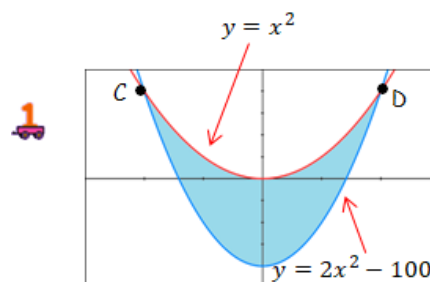
$$= \int_{-5}^5 -x^2 + 25 \, dx = \left[-\frac{x^3}{3} + 25x \right]_{-5}^5$$

$$= \left(-\frac{125}{3} + 125 \right) - \left(-\frac{125}{3} - 125 \right)$$


$$= \left(\frac{250}{3} \right) - \left(-\frac{250}{3} \right) = \frac{500}{3} \text{ units}^2$$


Gold questions


Calculate the following shaded areas between the curves...




Bronze Answers


 $\frac{99}{4} \text{ units}^2$


 $\frac{832}{3} \text{ units}^2$


 $\frac{117}{4} \text{ units}^2$


 Indefinite integral = -324
Shaded area = 324 units²

Silver Answers


 $\frac{275}{6} \text{ units}^2$


 $\frac{243}{2} \text{ units}^2$

 $\frac{160}{3} \text{ units}^2$

 8 units²

Gold Answers

 $\frac{4000}{3} \text{ units}^2$

 $\frac{256}{3} \text{ units}^2$