Outcome 1 – When given a factor

Bronze example

Examples...

Show that $(x+1)$ is a factor of $f(x) = x^3 - x^2 - 10x - 8$ and hence factorise it fully.

$(x+1)$ is a factor so $x = -1$

1. Find a root
2. Synthetic Division
3. Statement
4. Factorise the rest!

-1	1	-1	-10	-8
		-1	2	8
	1	-2	-8	0

Since no remainder, $(x+1)$ is a factor.

$$f(x) = (x+1)(x^2 - 2x - 8)$$
$$= (x+1)(x-4)(x+2)$$

Bronze questions

Fully factorise the following polynomials...

	Factor	Polynomial
1	$(x-1)$	$x^3 + 2x^2 - 13x + 10$
2	$(x+1)$	$x^3 - x^2 - 26x - 24$
3	$(x-2)$	$x^3 - 9x^2 + 6x + 16$
4	$(x+3)$	$x^3 - 4x^2 - 39x - 54$
5	$(x-5)$	$x^3 - 4x^2 - 7x + 10$
6	$(x+4)$	$x^3 + 3x^2 - 6x - 8$
7	$(x-6)$	$x^3 - 6x^2 - 4x + 24$
8	$(x-3)$	$x^3 + 4x^2 - 9x - 36$

Outcome 2 – When not given a factor/missing terms

Silver example

Examples...

Factorise fully $f(x) = x^3 - 7x - 6$.

$f(1) = 1 - 7 - 6 = -12$

$f(-1) = -1 + 7 - 6 = 0$

1. Find a root
2. Synthetic Division
3. Statement
4. Factorise the rest!

-1	1	0	-7	-6
		-1	1	6
	1	-1	-6	0

Since no remainder, $(x+1)$ is a factor.

$$f(x) = (x+1)(x^2 - x - 6)$$
$$= (x+1)(x-3)(x+2)$$

Silver questions

Fully factorise the following polynomials...

1. $f(x) = x^3 - 21x + 20$
2. $g(x) = 2x^3 + 3x^2 - 8x + 3$
3. $h(x) = x^3 - 111x + 110$
4. $f(x) = 4x^3 + 17x^2 + 16x + 3$
5. $h(x) = x^3 - 3x - 2$
6. $f(x) = 5x^3 - 19x^2 + 16x + 4$
7. $g(x) = x^3 - 12x - 16$
8. $h(x) = x^3 + x^2 - 12x$

Outcome 3 – Factorising polynomials of degree 4

Gold example

Examples... Factorise fully;

$f(x) = x^4 + x^3 - 31x^2 - x + 30$.

1. Find a root
2. Synthetic Division
3. Statement
4. Factorise the rest!
5. Synthetic Division AGAIN for the cubic bit!!

$f(1) = 1 + 1 - 31 - 1 + 30 = 0$

1	1	1	-31	-1	30
		1	2	-29	-30
	1	2	-29	-30	0

Since no remainder, $(x-1)$ is a factor.

$f(x) = (x-1)(x^3 + 2x^2 - 29x - 30)$ for the cubic bit!!

$f(-1) = -1 + 2 + 29 - 30 = 0$

-1	1	2	-29	-30
		-1	-1	30
	1	1	-30	0

Since no remainder, $(x+1)$ is a factor.

$$f(x) = (x-1)(x+1)(x^2 + x - 30)$$
$$= (x-1)(x+1)(x-5)(x+6)$$

Gold questions

Fully factorise the following polynomials...

1. $f(x) = x^4 - 5x^3 + 5x^2 + 5x - 6$
2. $g(x) = x^4 + x^3 - 31x^2 - x + 30$
3. $h(x) = 3x^4 - 13x^3 + x^2 + 13x - 4$
4. $f(x) = 2x^4 - 5x^3 - 5x^2 + 5x + 3$
5. $h(x) = x^4 - 23x^2 + 18x + 40$
6. $f(x) = x^4 - 3x^3 - 28x^2 + 36x + 144$

Bronze Answers

1. $(x-1)(x-2)(x+5)$
2. $(x+1)(x-6)(x+4)$
3. $(x-2)(x-8)(x+1)$
4. $(x+3)(x-9)(x+2)$
5. $(x-5)(x-1)(x+2)$
6. $(x+4)(x-2)(x+1)$
7. $(x-6)(x-2)(x+2)$
8. $(x-3)(x+3)(x+4)$

Silver Answers

1. $(x-1)(x-4)(x+5)$
2. $(x-1)(2x-1)(x+3)$
3. $(x-1)(x-10)(x+11)$
4. $(x+1)(4x+1)(x+3)$
5. $(x+1)(x+1)(x-2)$
6. $(x-2)(5x+1)(x-2)$
7. $(x+2)(x-4)(x+2)$
8. $x(x-3)(x+4)$

Gold Answers

1. $(x-1)(x+1)(x-2)(x-3)$
2. $(x-1)(x+1)(x-5)(x+6)$
3. $(x-1)(x+1)(3x-1)(x-4)$
4. $(x-1)(x+1)(2x+1)(x-3)$
5. $(x+1)(x-2)(x-4)(x+5)$
6. $(x+2)(x-3)(x-6)(x+4)$