

Outcome 1 - Finding the coordinates of stationary points

Bronze example

Example... **SP's occur when $f'(x) = 0$ **

Find the coordinates of the stationary points of the curve with equation $y = 2x^3 - 9x^2 + 12x + 5$.

$y = 2x^3 - 9x^2 + 12x + 5$
 $f'(x) = 6x^2 - 18x + 12$
 at SP's $f'(x) = 0$
 $6x^2 - 18x + 12 = 0$
 $6(x^2 - 3x + 2) = 0$
 $6(x-1)(x-2) = 0$
 $x = 1 \quad x = 2$

1. $f(x)$
 2. at SP's...
 3. Set = 0
 4. Find x
 5. Find y
 6. Answer question!

S.P.'s at
 (1, 10)
 and (2, 9)

$y = 2 - 9 + 12 + 5 = 10$
 $y = 16 - 36 + 24 + 5 = 9$

Bronze questions

Find the coordinates of the stationary point(s) of the following curves...

- 1 $y = x^2 - 2x + 5$
- 2 $y = x^3 + 3x^2 - 9x + 5$
- 3 $y = x^3 - 12x + 10$
- 4 $y = 2x^3 - 12x^2 + 11$
- 5 $y = 36x^2 - 3x^3$
- 6 $y = 3x^4 - 20x^3 + 30$

Outcome 2 - Determining the nature of stationary points

Silver example

Example... **SP's occur when $f'(x) = 0$ **

A curve with equation $y = x^3 + 3x^2 - 9x + 7$ has stationary points at $(-3, 34)$ and $(1, 2)$. Determine their nature.

$y = x^3 + 3x^2 - 9x + 7$
 $f'(x) = 3x^2 + 6x - 9$

Draw a Nature Table!

Put in your stationary values

x	-10	-3	0	1	10
$f'(x)$	+ve	0	-ve	0	+ve
slope	/	-	\	-	/

Max T.P. at $(-3, 34)$ Min T.P. at $(1, 2)$

Silver questions

Determine the nature of the stationary points of the following curves...

- 1 $y = x^2 - 2x + 5$ → SP at $(1, 4)$
- 2 $y = x^3 + 3x^2 - 9x + 5$ → SP's at $(-3, 32)$ and $(1, 0)$
- 3 $y = x^3 - 12x + 10$ → SP's at $(-2, 27)$ and $(2, -5)$
- 4 $y = 2x^3 - 12x^2 + 11$ → SP's at $(0, 11)$ and $(4, -53)$
- 5 $y = 36x^2 - 3x^3$ → SP's at $(0, 0)$ and $(8, 768)$
- 6 $y = 3x^4 - 20x^3 + 30$ → SP's at $(0, 30)$ and $(5, -595)$

Outcome 3 - Curve sketching

Gold example

****Sub in $x = 0$ **** 1. Calculate the y-intercept

Sketch the graph of $y = x^3 - 3x^2$ showing clearly where it meets the x and y axes.

2. Find the roots $x^3 - 3x^2 = 0$ (0, 0)
 $x^2(x-3) = 0$ and (3, 0)
 $x = 0 \quad x = 3$

3. Calculate stationary points and determine their nature.

$y = x^3 - 3x^2$
 $f'(x) = 3x^2 - 6x$
 @ SP's $f'(x) = 0$
 $3x^2 - 6x = 0$
 $3x(x-2) = 0$
 $x = 0 \quad x = 2$
 $y = 0 - 0 = 0$
 $y = 8 - 12 = -4$

Max TP at (0, 0) Min TP at (2, -4)

4. Show ALL this information in a sketch

Gold questions

Sketch the following curves showing intersection with both axes and any stationary points...

- 1 $y = 2x^3 - 18x^2$
- 2 $y = 6x^2 - 20x^3$
- 3 $y = (x+5)(x-1)^2$
- 4 $y = (x-6)(x-1)(x+2)$

Bronze Answers

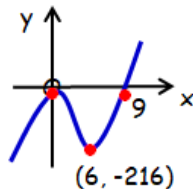
1. (1, 4)
2. (-3, 32) and (1, 0)
3. (-2, 27) and (2, -5)
4. (0, 11) and (4, -53)
5. (0, 0) and (8, 768)
6. (0, 30) and (5, -595)

Silver Answers

1. Min TP at (1, 4)
2. Max TP at (-3, 32), Min TP at (1, 0)
3. Max TP at (-2, 27), Min TP at (2, -5)
4. Max TP at (0, 11), Min TP at (4, -53)
5. Min TP at (0, 0), Max TP at (8, 768)
6. Falling pt of inflection at (0, 30),
Min TP at (5, -595)

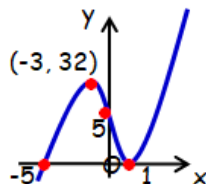
Gold Answers

1. Y-intercept = (0, 0)
Roots = (0, 0) & (9, 0)
TP's = Max (0, 0)
& min (6, -216)



2. Y-intercept = (0, 0)
Roots = (0, 0) & $(\frac{3}{10}, 0)$
TP's = Max (0, 0)
& min $(\frac{1}{5}, \frac{2}{25})$

3. Y-intercept = (0, 5)
Roots = (-5, 0) & (1, 0)
TP's = Max (-3, 32)
& min (1, 0)



4. Y-intercept = (0, 5)
Roots = (-5, 0) & (1, 0)
TP's = Max (-3, 0)
& min (1, -216)

