Higher Mathematics

2016 Paper 2

Time allowed = 1 hr 30 mins Marks available = 70

For each question, you can scan the QR codes if using a paper copy or click on the links viewing this document electronically. This will allow you to view the worked solutions for each question. You can also either scan this QR Code or click on the link below to view this paper's marking scheme;

https://www.sqa.org.uk/pastpapers/papers/instructions/2016/mi_NH_Mathematics_all_2016.pdf

Remember to record your percentage for this paper in your analysis grid (your score ÷ 70 × 100).

FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

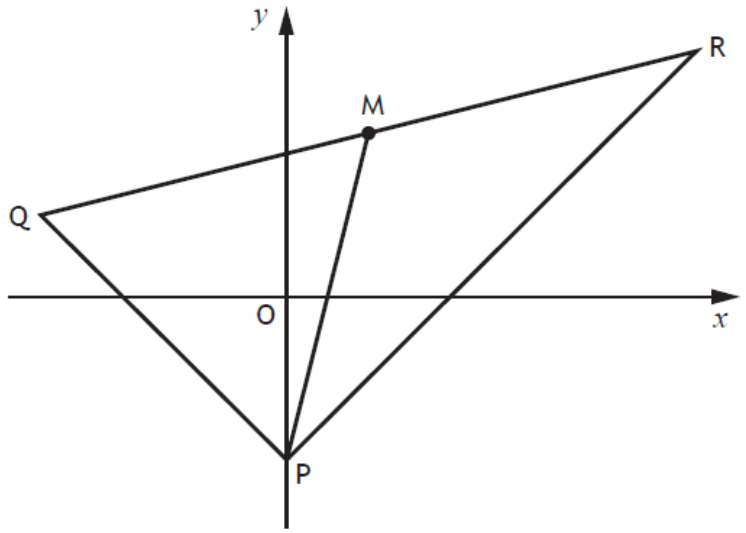
Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Attempt ALL questions

Total marks — 70

1. PQR is a triangle with vertices $P(0,-4)$, $Q(-6,2)$ and $R(10,6)$.



(a) (i) State the coordinates of M, the midpoint of QR. 1

(ii) Hence find the equation of PM, the median through P. 2

(b) Find the equation of the line, L, passing through M and perpendicular to PR. 3

(c) Show that line L passes through the midpoint of PR. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/WSIKboZrbBI

Video Lessons: 1·8 Bronze Outcome 1, 1·6 Silver Outcome 2,

1·9 Silver Outcome 2

2. Find the range of values for p such that $x^2 - 2x + 3 - p = 0$ has no real roots. 3

Scan the QR code or click on the link to view the worked solutions;

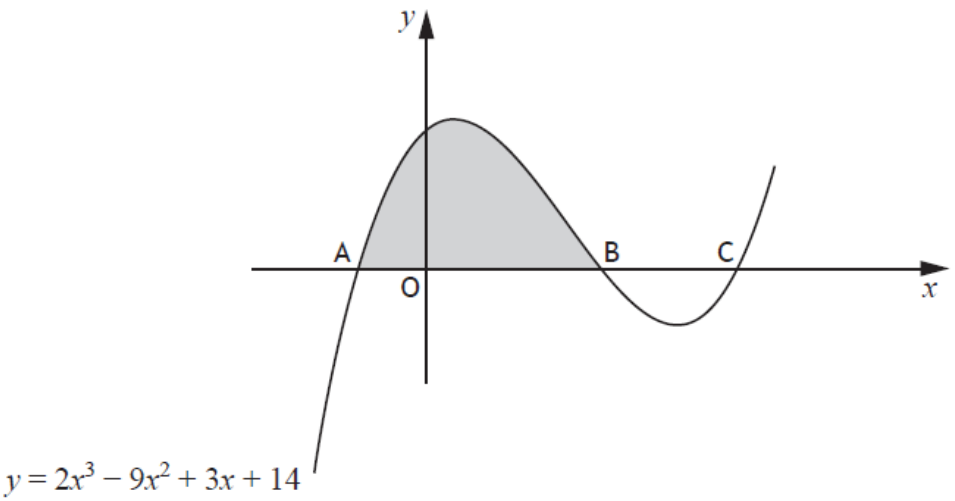
https://youtu.be/0ITONUltLDY

Video Lesson: 8·4 Gold Outcome 3

3. (a) (i) Show that $(x+1)$ is a factor of $2x^3 - 9x^2 + 3x + 14$. 2

(ii) Hence solve the equation $2x^3 - 9x^2 + 3x + 14 = 0$. 3

(b) The diagram below shows the graph with equation $y = 2x^3 - 9x^2 + 3x + 14$.
The curve cuts the x-axis at A, B and C.



(i) Write down the coordinates of the points A and B. 1

(ii) Hence calculate the shaded area in the diagram. 4

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/rOeP5reNQVw

Video Lessons: 7·1 Bronze Outcome 1, 9·4 Gold Outcome 3

4. Circles C_1 and C_2 have equations $(x+5)^2+(y-6)^2=9$ and $x^2+y^2-6x-16=0$ respectively.

   (a) Write down the centres and radii of C_1 and C_2. 4

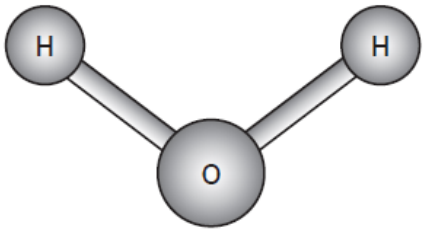
   (b) Show that C_1 and C_2 do not intersect. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/l_Tq52GWwhA

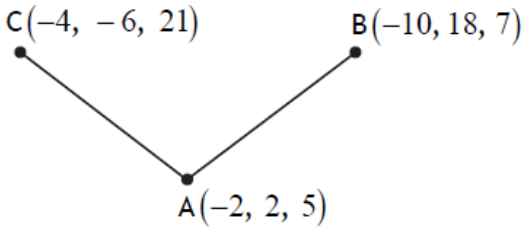
Video Lesson: 11·4 Gold Outcome 3

5. The picture shows a model of a water molecule.



Relative to suitable coordinate axes, the oxygen atom is positioned at point $A(-2,\ 2,\ 5)$.

The two hydrogen atoms are positioned at points $B(-10,\ 18,\ 7)$ and $C(-4,\ -6,\ 21)$ as shown in the diagram below.



   (a) Express $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in component form. 2

   (b) Hence, or otherwise, find the size of angle BAC. 4

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/N_ihTh2Z2I4

Video Lesson: 12·4 Outcome 1

6. Scientists are studying the growth of a strain of bacteria. The number of bacteria present is given by the formula

$$B(t) = 200e^{0\cdot107t},$$

where t represents the number of hours since the study began.

(a) State the number of bacteria present at the start of the study. 1

(b) Calculate the time taken for the number of bacteria to double. 4

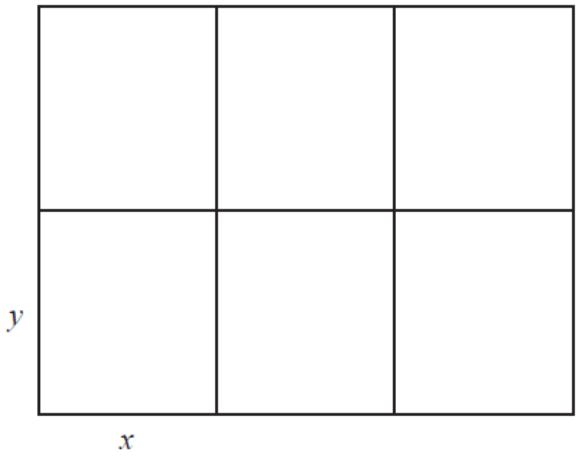
Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/HbHphT_mJbs

Video Lessons: 14·3 Gold Outcome 3

7. A council is setting aside an area of land to create six fenced plots where local residents can grow their own food.

Each plot will be a rectangle measuring x metres by y metres as shown in the diagram.



(a) The area of land being set aside is $108\ \text{m}^2$.

Show that the total length of fencing, L metres, is given by

$$L(x) = 9x + \frac{144}{x}.$$ 3

(b) Find the value of x that minimises the length of fencing required. 6

Scan the QR code or click on it to view the worked solutions;

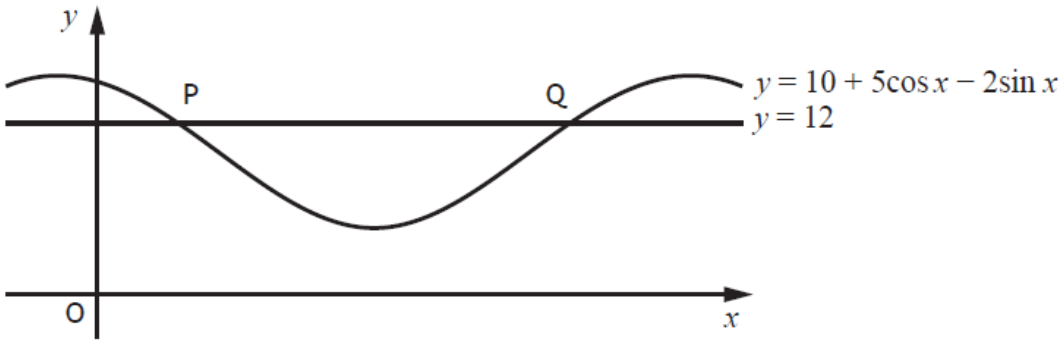
https://youtu.be/HSZ0t27O8-M

Video Lesson: 6·8 Outcome 1

8. (a) Express $5\cos x - 2\sin x$ in the form $k\cos(x + a)$, where $k > 0$ and $0 < a < 2\pi$. 4

(b) The diagram shows a sketch of part of the graph of $y = 10 + 5\cos x - 2\sin x$ and the line with equation $y = 12$.

The line cuts the curve at the points P and Q.



Find the x-coordinates of P and Q. 4

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/krhaluWUBeQ

Video Lessons: 15·1 Gold Outcome 3, 15·2 Bronze Outcome 1

9. For a function f, defined on a suitable domain, it is known that:

- $f'(x) = \dfrac{2x+1}{\sqrt{x}}$
- $f(9) = 40$

Express $f(x)$ in terms of x. 4

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/vzDVNCJku9U

Video Lesson: 9·3 Outcome 1

10. (a) Given that $y = (x^2+7)^{\frac{1}{2}}$, find $\dfrac{dy}{dx}$. 2

(b) Hence find $\displaystyle\int \frac{4x}{\sqrt{x^2+7}}\,dx$. 1

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/7b2C7v-GksE

Video Lesson: 13·1 Bronze Outcome 1

11. (a) Show that $\sin 2x \tan x = 1 - \cos 2x$, where $\frac{\pi}{2} < x < \frac{3\pi}{2}$. 4

(b) Given that $f(x) = \sin 2x \tan x$, find $f'(x)$. 2

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/2_ayqI3NlMw

Video Lessons: 10·3 Gold Outcome 3, 13·1 Silver Outcome 2

[END OF QUESTION PAPER]