Higher Mathematics

2016 Paper 1

Time allowed = 1 hr 10 mins Marks available = 60

For each question, you can scan the QR codes if using a paper copy or click on the links viewing this document electronically. This will allow you to view the worked solutions for each question. You can also either scan this QR Code or click on the link below to view this paper's marking scheme;

https://www.sqa.org.uk/pastpapers/papers/instructions/2016/mi_NH_Mathematics_all_2016.pdf

Remember to record your percentage for this paper in your analysis grid (your score ÷ 60 × 100).

FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Attempt ALL questions

Total marks — 60

1. Find the equation of the line passing through the point $(-2, 3)$ which is parallel to the line with equation $y + 4x = 7$. 2

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/JWtKlv5UO2A

Video Lesson: 1·6 Bronze Outcome 1

2. Given that $y = 12x^3 + 8\sqrt{x}$, where $x > 0$, find $\frac{dy}{dx}$. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/Ne_43gWbUUU

Video Lesson: 6·1 Silver Outcome 2

3. A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 10$ with $u_3 = 6$.

   (a) Find the value of u_4. 1

   (b) Explain why this sequence approaches a limit as $n \to \infty$. 1

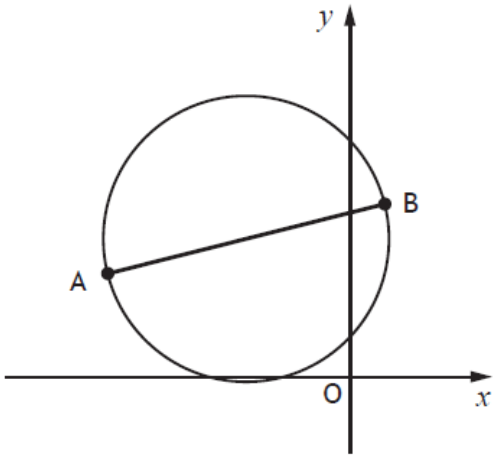
   (c) Calculate this limit. 2

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/EpYEvXmS3rA

Video Lessons: 2·1 Silver Outcome 2, 2·2 Silver Outcome 2

4. A and B are the points $(-7, 3)$ and $(1, 5)$.
   AB is a diameter of a circle.



   Find the equation of this circle. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/pWl0SZ4lHQs

Video Lesson: 11·1 Gold Outcome 3

5. Find $\int 8\cos(4x+1)\,dx$. 2

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/V1yM9xZczec

Video Lesson: 13·2 Silver Outcome 2

6. Functions f and g are defined on $\mathbb{R}$, the set of real numbers.
The inverse functions f^{-1} and g^{-1} both exist.

(a) Given $f(x)=3x+5$, find $f^{-1}(x)$. 3

(b) If $g(2)=7$, write down the value of $g^{-1}(7)$. 1

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/W784vJnGBUM

Video Lesson: 3·3 Outcome 1

7. Three vectors can be expressed as follows:

$$\overrightarrow{FG} = -2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$$

$$\overrightarrow{GH} = 3\mathbf{i} + 9\mathbf{j} - 7\mathbf{k}$$

$$\overrightarrow{EH} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$$

(a) Find $\overrightarrow{FH}$. 2

(b) Hence, or otherwise, find $\overrightarrow{FE}$. 2

Scan the QR code or click on it to view the worked solutions;

https://youtu.be/oKTF8mJViIc

Video Lesson: 12·1 Gold Outcome 3

8. Show that the line with equation $y = 3x - 5$ is a tangent to the circle with equation $x^2 + y^2 + 2x - 4y - 5 = 0$ and find the coordinates of the point of contact. 5

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/qV4FNBX-pRc

Video Lesson: 11·3 Silver Outcome 2

9. (a) Find the x-coordinates of the stationary points on the graph with equation $y = f(x)$, where $f(x) = x^3 + 3x^2 - 24x$. 4

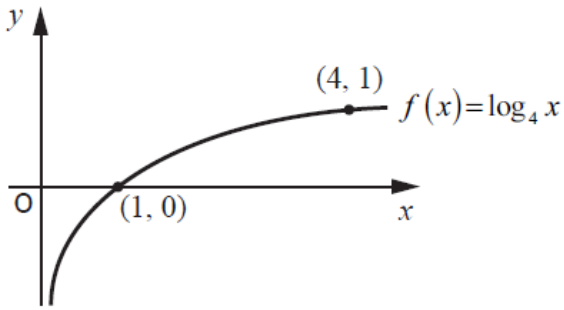
(b) Hence determine the range of values of x for which the function f is strictly increasing. 2

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/8cVj4Q1V7X8

Video Lessons: 6·5 Bronze Outcome 1, 6·4 Bronze Outcome 1

10. The diagram below shows the graph of the function $f(x) = \log_4 x$, where $x > 0$.



The inverse function, f^{-1}, exists.

On the diagram in your answer booklet, sketch the graph of the inverse function. 2

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/RLM-G_BsFLc

Video Lesson: 4·2 Gold Outcome 3

11. (a) A and C are the points $(1, 3, -2)$ and $(4, -3, 4)$ respectively.
Point B divides AC in the ratio $1 : 2$.
Find the coordinates of B. 2

(b) $k\overrightarrow{AC}$ is a vector of magnitude 1, where $k > 0$.
Determine the value of k. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/mK4kW8__8I4

Video Lessons: 12·2 Gold Outcome 3

12. The functions f and g are defined on $\mathbb{R}$, the set of real numbers by $f(x) = 2x^2 - 4x + 5$ and $g(x) = 3 - x$.

(a) Given $h(x) = f(g(x))$, show that $h(x) = 2x^2 - 8x + 11$. 2

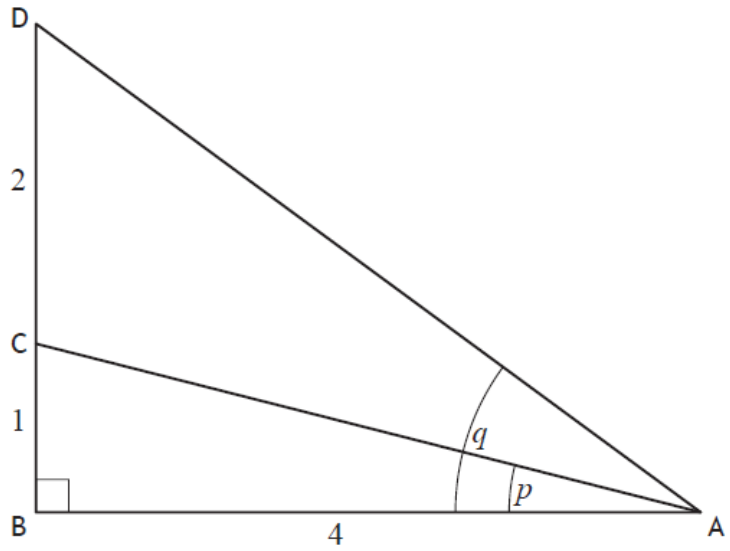
(b) Express $h(x)$ in the form $p(x+q)^2 + r$. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/Kvf_apWRE80

Video Lessons: 3·2 Silver Outcome 2, 8·2 Bronze Outcome 1

13. Triangle ABD is right-angled at B with angles BAC $= p$ and BAD $= q$ and lengths as shown in the diagram below.



Show that the exact value of $\cos(q-p)$ is $\frac{19\sqrt{17}}{85}$. 5

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/d-BshlbvqyE

Video Lesson: 10·1 Gold Outcome 3

14. (a) Evaluate $\log_5 25$. 1

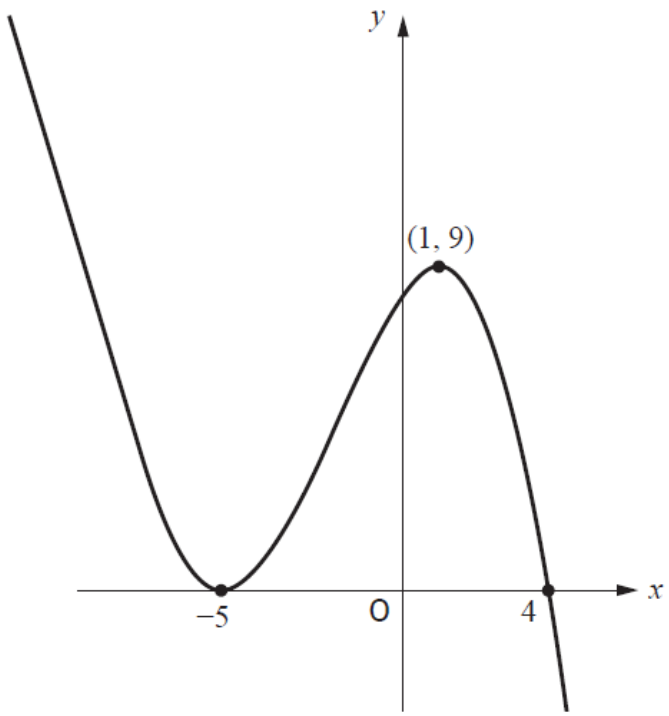
(b) Hence solve $\log_4 x + \log_4(x-6) = \log_5 25$, where $x > 6$. 5

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/ZFgINjUKTOg

Video Lessons: 14·1 Bronze Outcome 1, 14·2 Silver Outcome 2

15. The diagram below shows the graph with equation $y = f(x)$, where $f(x) = k(x-a)(x-b)^2$.



(a) Find the values of a, b and k. 3

(b) For the function $g(x) = f(x) - d$, where d is positive, determine the range of values of d for which $g(x)$ has exactly one real root. 1

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/OxSyXPEmodA

Video Lesson: 8·1 Gold Outcome 3

[END OF QUESTION PAPER]