Higher Mathematics

2018 Paper 2

Time allowed = 1 hr 30 mins Marks available = 70

For each question, you can scan the QR codes if using a paper copy or click on the links viewing this document electronically. This will allow you to view the worked solutions for each question. You can also either scan this QR Code or click on the link below to view this paper's marking scheme;

https://www.sqa.org.uk/pastpapers/papers/instructions/2018/mi_NH_Mathematics_all_2018.pdf

Remember to record your percentage for this paper in your analysis grid (your score ÷ 70 × 100).

FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

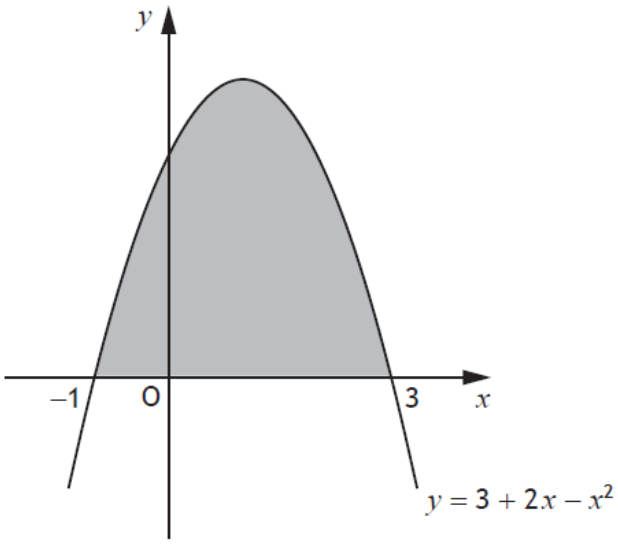
Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Attempt ALL questions

Total marks — 70

1. The diagram shows the curve with equation $y = 3 + 2x - x^2$.



Calculate the shaded area. 4

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/9Li_Hf6prec

Video Lesson: 9·4 Bronze Outcome 1

2. Vectors $\mathbf{u}$ and $\mathbf{v}$ are defined by $\mathbf{u}=\begin{pmatrix}-1\\4\\-3\end{pmatrix}$ and $\mathbf{v}=\begin{pmatrix}-7\\8\\5\end{pmatrix}$.

(a) Find $\mathbf{u}.\mathbf{v}$. 1

(b) Calculate the acute angle between $\mathbf{u}$ and $\mathbf{v}$. 4

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/w84YNIf4MK4

Video Lesson: 12·4 Outcome 1

3. A function, f, is defined on the set of real numbers by $f(x)=x^3-7x-6$.

Determine whether f is increasing or decreasing when $x=2$. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/Km8Nw7VOd90

Video Lesson: 6·4 Bronze Outcome 1

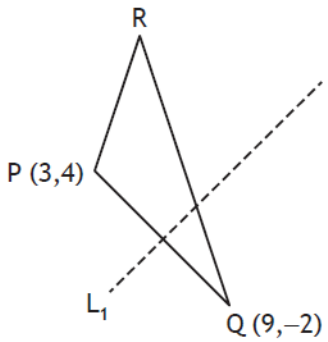
4. Express $-3x^2-6x+7$ in the form $a(x+b)^2+c$. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/6k47TxRYLz4

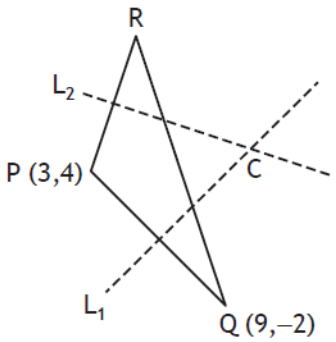
Video Lesson: 8·2 Silver Outcome 2

5. PQR is a triangle with P(3,4) and Q(9,−2).



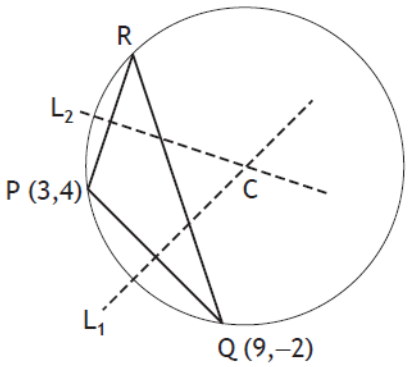
(a) Find the equation of L_1, the perpendicular bisector of PQ. 3

The equation of L_2, the perpendicular bisector of PR is $3y + x = 25$.



(b) Calculate the coordinates of C, the point of intersection of L_1 and L_2. 2

C is the centre of the circle which passes through the vertices of triangle PQR.



(c) Determine the equation of this circle. 2

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/Sv6m62s8sFg

Video Lessons: 1·8 Gold Outcome 3, 1·9 Gold Outcome 3,

11·1 Bronze Outcome 1

SQA Pastpapers

6. Functions, f and g, are given by $f(x) = 3 + \cos x$ and $g(x) = 2x$, $x \in \mathbb{R}$.

(a) Find expressions for

(i) $f(g(x))$ and 2

(ii) $g(f(x))$. 1

(b) Determine the value(s) of x for which $f(g(x)) = g(f(x))$ where $0 \leq x < 2\pi$. 6

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/fX4scQuzZ2k

Video Lesson: 3·2 Silver Outcome 2, 10·2 Gold Outcome 3

7. (a) (i) Show that $(x-2)$ is a factor of $2x^3 - 3x^2 - 3x + 2$. 2

(ii) Hence, factorise $2x^3 - 3x^2 - 3x + 2$ fully. 2

The fifth term, u_5, of a sequence is $u_5 = 2a - 3$.

The terms of the sequence satisfy the recurrence relation $u_{n+1} = au_n - 1$.

(b) Show that $u_7 = 2a^3 - 3a^2 - a - 1$. 1

For this sequence, it is known that

- $u_7 = u_5$
- a limit exists.

(c) (i) Determine the value of a. 3

(ii) Calculate the limit. 1

Scan the QR code or click on it to view the worked solutions;

https://youtu.be/zEzMZUfCQws

Video Lesson: 7·1 Bronze Outcome 1, 2·2 Silver Outcome 2

8. (a) Express $2\cos x° - \sin x°$ in the form $k\cos(x-a)°$, $k>0$, $0<a<360$. 4

(b) Hence, or otherwise, find

(i) the minimum value of $6\cos x° - 3\sin x°$ and 1

(ii) the value of x for which it occurs where $0 \leq x < 360$. 2

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/CasNvFUNP2w

Video Lessons: 15·1 Silver Outcome 2, 15·2 Silver Outcome 2

9. A sector with a particular fixed area has radius x cm.

The perimeter, P cm, of the sector is given by

$$P = 2x + \frac{128}{x}.$$

Find the minimum value of P. 6

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/RXz2RtJQ2qQ

Video Lesson: 6·8 Outcome 1

10. The equation $x^2+(m-3)x+m=0$ has two real and distinct roots.
Determine the range of values for m. 4

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/maeXBNiztp8

Video Lessons: 8·4 Gold Outcome 3, 8·3 Silver Outcome 2

11. A supermarket has been investigating how long customers have to wait at the checkout.
During any half hour period, the percentage, $P\%$, of customers who wait for less than t minutes, can be modelled by

$$P=100\left(1-e^{kt}\right), \text{ where } k \text{ is a constant.}$$

(a) If 50% of customers wait for less than 3 minutes, determine the value of k. 4

(b) Calculate the percentage of customers who wait for 5 minutes or longer. 2

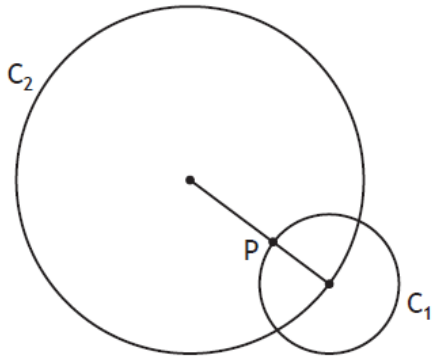
Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/M7mT5ctVxaY

Video Lesson: 14·3 Gold Outcome 3

12. Circle C_1 has equation $(x-13)^2+(y+4)^2=100$.

Circle C_2 has equation $x^2+y^2+14x-22y+c=0$.



(a) (i) Write down the coordinates of the centre of C_1. 1

(ii) The centre of C_1 lies on the circumference of C_2.

Show that $c=-455$. 1

The line joining the centres of the circles intersects C_1 at P.

(b) (i) Determine the ratio in which P divides the line joining the centres of the circles. 2

(ii) Hence, or otherwise, determine the coordinates of P. 2

P is the centre of a third circle, C_3.

C_2 touches C_3 internally.

(c) Determine the equation of C_3. 1

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/5iofjjnDIMQ

[END OF QUESTION PAPER]