Higher Mathematics

2017 Paper 1

Time allowed = 1 hr 10 mins Marks available = 60

For each question, you can click below to view the worked solutions for each question. You can also view this paper's marking scheme below;

https://www.sqa.org.uk/pastpapers/papers/instructions/2017/mi_NH_Mathematics_all_2017.pdf

Remember to record your percentage for this paper in your analysis grid (your score ÷ 60 × 100).

FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Attempt ALL questions

Total marks — 60

1. Functions f and g are defined on suitable domains by $f(x)=5x$ and $g(x)=2\cos x$.

   (a) Evaluate $f(g(0))$. 1

   (b) Find an expression for $g(f(x))$. 2

Click here to view the worked solutions.

Video Lesson: 3·2 Silver Outcome 2

2. The point P(–2, 1) lies on the circle $x^2+y^2-8x-6y-15=0$.
   Find the equation of the tangent to the circle at P. 4

Click here to view the worked solutions.

Video Lesson: 11·2 Silver Outcome 2

3. Given $y=(4x-1)^{12}$, find $\frac{dy}{dx}$. 2

Click here to view the worked solutions.

Video Lesson: 13·1 Bronze Outcome 1

4. Find the value of k for which the equation $x^2+4x+(k-5)=0$ has equal roots. 3

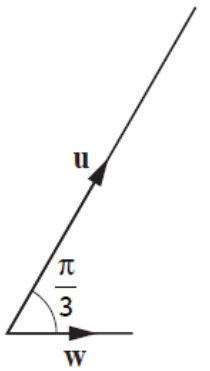
Click here to view the worked solutions.

Video Lesson: 8·4 Gold Outcome 3

5. Vectors $\mathbf{u}$ and $\mathbf{v}$ are $\begin{pmatrix}5\\1\\-1\end{pmatrix}$ and $\begin{pmatrix}3\\-8\\6\end{pmatrix}$ respectively.

(a) Evaluate $\mathbf{u}.\mathbf{v}$. 1

(b)



Vector $\mathbf{w}$ makes an angle of $\frac{\pi}{3}$ with $\mathbf{u}$ and $|\mathbf{w}|=\sqrt{3}$.

Calculate $\mathbf{u}.\mathbf{w}$. 3

Click here to view the worked solutions.

Video Lesson: 12·4 Outcome 1

6. A function, h, is defined by $h(x) = x^3 + 7$, where $x \in \mathbb{R}$.
Determine an expression for $h^{-1}(x)$. 3

Click here to view the worked solutions.

Video Lesson: 3·3 Outcome 1

7. A $(-3, 5)$, B $(7, 9)$ and C $(2, 11)$ are the vertices of a triangle.
Find the equation of the median through C. 3

Click here to view the worked solutions.

Video Lesson: 1·8 Bronze Outcome 1

8. Calculate the rate of change of $d(t) = \frac{1}{2t}$, $t \neq 0$, when $t = 5$. 3

Click here to view the worked solutions.

Video Lesson: 6·2 Silver Outcome 2

9. A sequence is generated by the recurrence relation $u_{n+1} = mu_n + 6$ where m is a constant.

(a) Given $u_1 = 28$ and $u_2 = 13$, find the value of m. 2

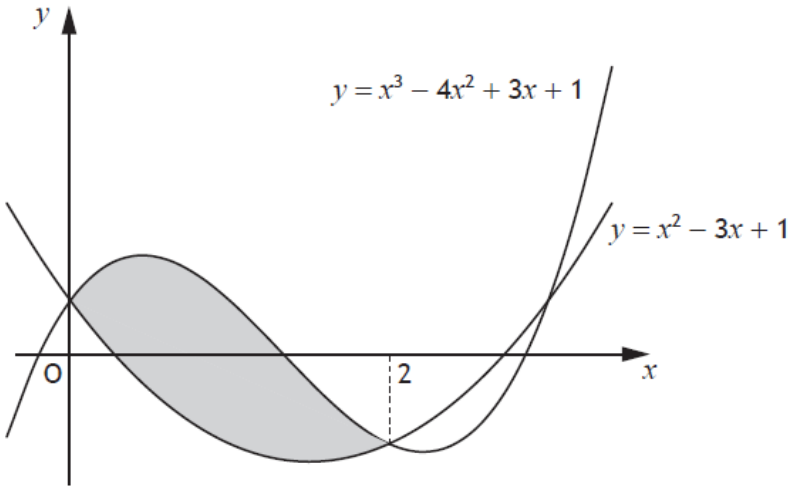
(b) (i) Explain why this sequence approaches a limit as $n \to \infty$. 1

(ii) Calculate this limit. 2

Click here to view the worked solutions.

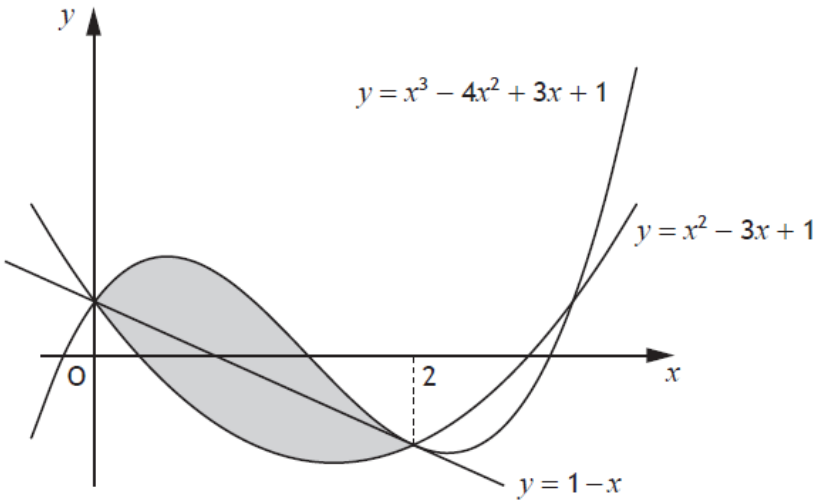
Video Lesson: 2·2 Silver Outcome 2

10. Two curves with equations $y = x^3 - 4x^2 + 3x + 1$ and $y = x^2 - 3x + 1$ intersect as shown in the diagram.



(a) Calculate the shaded area. **5**

The line passing through the points of intersection of the curves has equation $y = 1 - x$.



(b) Determine the fraction of the shaded area which lies below the line $y = 1 - x$. **4**

Click here to view the worked solutions.

Video Lesson: 9·4 Gold Outcome 3

11. A and B are the points $(-7, 2)$ and $(5, a)$.
AB is parallel to the line with equation $3y - 2x = 4$.
Determine the value of a. 3

Click here to view the worked solutions.

Video Lessons: 1·1 Gold Outcome 3, 1·6 Bronze Outcome 1

12. Given that $\log_a 36 - \log_a 4 = \frac{1}{2}$, find the value of a. 3

Click here to view the worked solutions.

Video Lessons: 14·2 Silver Outcome 2

13. Find $\int \frac{1}{(5-4x)^{\frac{1}{2}}}dx, \quad x<\frac{5}{4}$. 4

Click here to view the worked solutions.

Video Lesson: 13·2 Silver Outcome 2

14. (a) Express $\sqrt{3}\sin x° - \cos x°$ in the form $k\sin(x-a)°$, where $k>0$ and $0<a<360$. 4

(b) Hence, or otherwise, sketch the graph with equation $y=\sqrt{3}\sin x° - \cos x°$, $0 \le x \le 360$. 3

Use the diagram provided in the answer booklet.

Click here to view the worked solutions.

Video Lessons: 15·1 Bronze Outcome 1, 15·2 Gold Outcome 3

15. A quadratic function, f, is defined on $\mathbb{R}$, the set of real numbers.

Diagram 1 shows part of the graph with equation $y = f(x)$.
The turning point is $(2, 3)$.

Diagram 2 shows part of the graph with equation $y = h(x)$.
The turning point is $(7, 6)$.

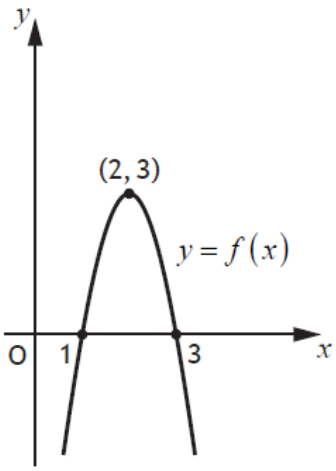


Diagram 1

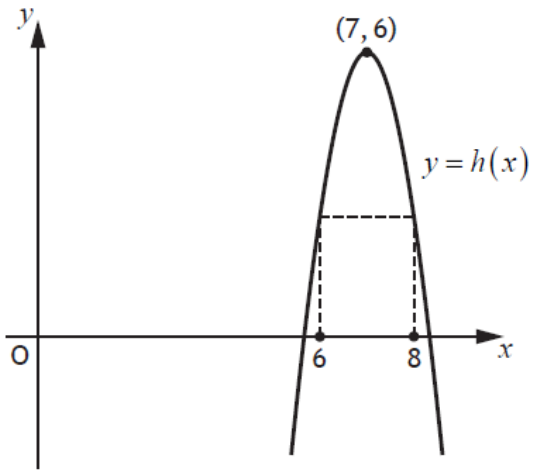


Diagram 2

(a) Given that $h(x) = f(x + a) + b$.

Write down the values of a and b. **2**

(b) It is known that $\int_1^3 f(x)\,dx = 4$.

Determine the value of $\int_6^8 h(x)\,dx$. **1**

(c) Given $f'(1) = 6$, state the value of $h'(8)$. **1**

Click here to view the worked solutions.

[END OF QUESTION PAPER]