Higher Mathematics

2017 Paper 2

Time allowed = 1 hr 30 mins   Marks available = 70

For each question, you can click below to view the worked solutions for each question. You can also view this paper's marking scheme below;

https://www.sqa.org.uk/pastpapers/papers/instructions/2017/mi_NH_Mathematics_all_2017.pdf

Remember to record your percentage for this paper in your analysis grid (your score ÷ 70 × 100).

FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

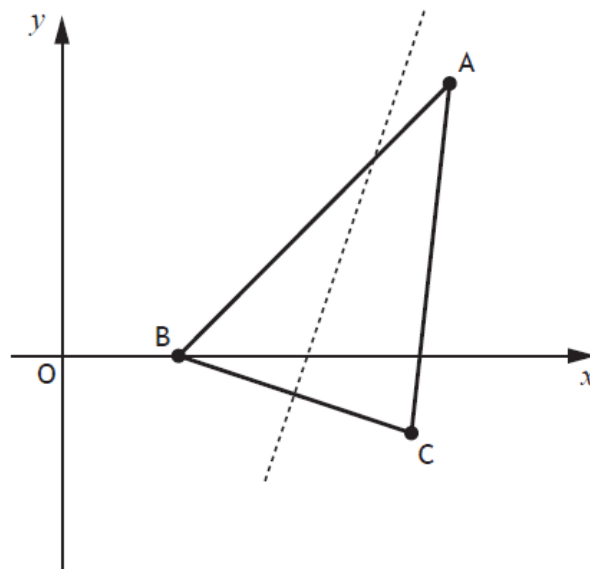**Table of standard integrals**

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\dfrac{1}{a}\cos ax + c$
$\cos ax$	$\dfrac{1}{a}\sin ax + c$

Attempt ALL questions
Total marks — 70

1. Triangle ABC is shown in the diagram below.

    The coordinates of B are (3,0) and the coordinates of C are (9,−2).

    The broken line is the perpendicular bisector of BC.

    (a) Find the equation of the perpendicular bisector of BC.   4

    (b) The line AB makes an angle of 45° with the positive direction of the x-axis.
         Find the equation of AB.   2

    (c) Find the coordinates of the point of intersection of AB and the perpendicular bisector of BC.   2
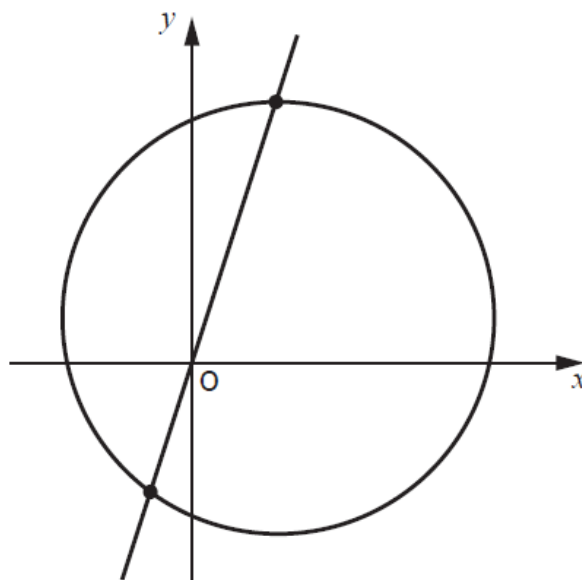
Click here to view the video solutions.

Video Lessons: 1·8 Gold Outcome 3, 1·3 Bronze Outcome 1, 1·9 Silver Outcome 2

2. (a) Show that $(x-1)$ is a factor of $f(x) = 2x^3 - 5x^2 + x + 2$.   2

    (b) Hence, or otherwise, solve $f(x) = 0$.   3

Click here to view the video solutions.

Video Lesson: 7·1 Bronze Outcome 1

3. The line $y = 3x$ intersects the circle with equation $(x-2)^2 + (y-1)^2 = 25$.

    Find the coordinates of the points of intersection.   5
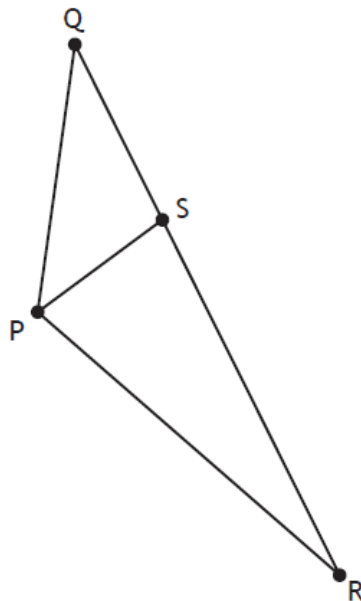
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Video Lesson: 11·3 Bronze Outcome 1

4. (a) Express $3x^2 + 24x + 50$ in the form $a(x+b)^2 + c$.   **3**

    (b) Given that $f(x) = x^3 + 12x^2 + 50x - 11$, find $f'(x)$.   **2**

    (c) Hence, or otherwise, explain why the curve with equation $y = f(x)$ is strictly
         increasing for all values of x.   **2**

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Video Lesson: 8·2 Bronze Outcome 1

5. In the diagram, $\overrightarrow{PR} = 9\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{RQ} = -12\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$.

    (a) Express $\overrightarrow{PQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$.   **2**

    The point S divides QR in the ratio 1:2.

    (b) Show that $\overrightarrow{PS} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$.   **2**

    (c) Hence, find the size of angle QPS.   **5**

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Video Lesson: 12·4 Outcome 1

SQA Pastpapers

6. Solve $5\sin x - 4 = 2\cos 2x$ for $0 \le x < 2\pi$.   **5**

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Video Lessons: 10·2 Gold Outcome 3

7. (a) Find the x-coordinate of the stationary point on the curve

   with equation $y = 6x - 2\sqrt{x^3}$.   **4**

   (b) Hence, determine the greatest and least values of y in the interval $1 \le x \le 9$.   **3**

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Video Lesson: 6·6 Outcome 1

8. Sequences may be generated by recurrence relations of the form
    $u_{n+1} = k u_n - 20$, $u_0 = 5$ where $k \in \mathbb{R}$.

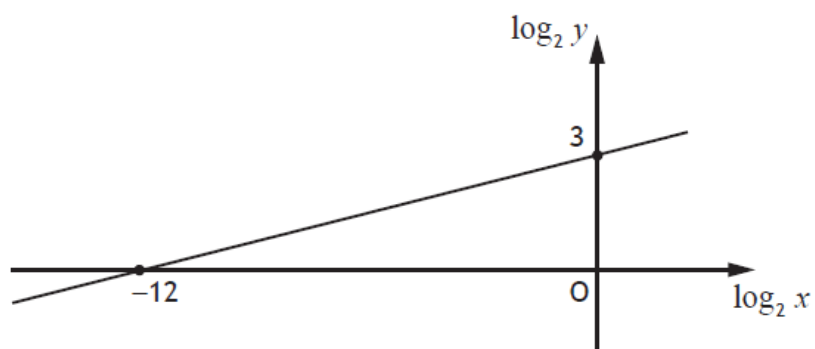   (a) Show that $u_2 = 5k^2 - 20k - 20$.   2

    (b) Determine the range of values of k for which $u_2 < u_0$.   4

Click here to view the video solutions.

Video Lessons: 2·1 Silver Outcome 2, 8·3 Silver Outcome 2

9. Two variables, x and y, are connected by the equation $y = kx^n$.

    The graph of $\log_2 y$ against $\log_2 x$ is a straight line as shown.
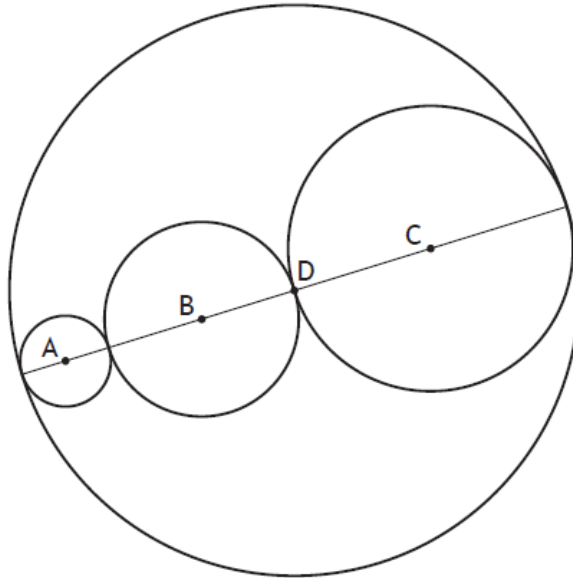

    Find the values of k and n.   5

Click here to view the video solutions.

Video Lesson: 14·4 Gold Outcome 3

10. (a) Show that the points A(−7, −2), B(2, 1) and C(17, 6) are collinear.   **3**

Three circles with centres A, B and C are drawn inside a circle with centre D as shown.

The circles with centres A, B and C have radii r_A, r_B and r_C respectively.

- $r_A = \sqrt{10}$

- $r_B = 2r_A$

- $r_C = r_A + r_B$

(b) Determine the equation of the circle with centre D.   **4**

Click <u>here</u> to view the video solutions.

Video Lesson: 1·7 Outcome 1

11. (a) Show that $\dfrac{\sin 2x}{2\cos x} - \sin x \cos^2 x = \sin^3 x$, where $0 < x < \dfrac{\pi}{2}$.   **3**

   (b) Hence, differentiate $\dfrac{\sin 2x}{2\cos x} - \sin x \cos^2 x$, where $0 < x < \dfrac{\pi}{2}$.   **3**

Click here to view the video solutions.

Video Lessons: 10·3 Gold Outcome 3, 13·1 Gold Outcome 3

[END OF QUESTION PAPER]