Higher Mathematics

2018 Paper 1

Time allowed = 1 hr 10 mins Marks available = 60

For each question, you can scan the QR codes if using a paper copy or click on the links viewing this document electronically. This will allow you to view the worked solutions for each question. You can also either scan this QR Code or click on the link below to view this paper's marking scheme;

https://www.sqa.org.uk/pastpapers/papers/instructions/2018/mi_NH_Mathematics_all_2018.pdf

Remember to record your percentage for this paper in your analysis grid (your score ÷ 60 × 100).

FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

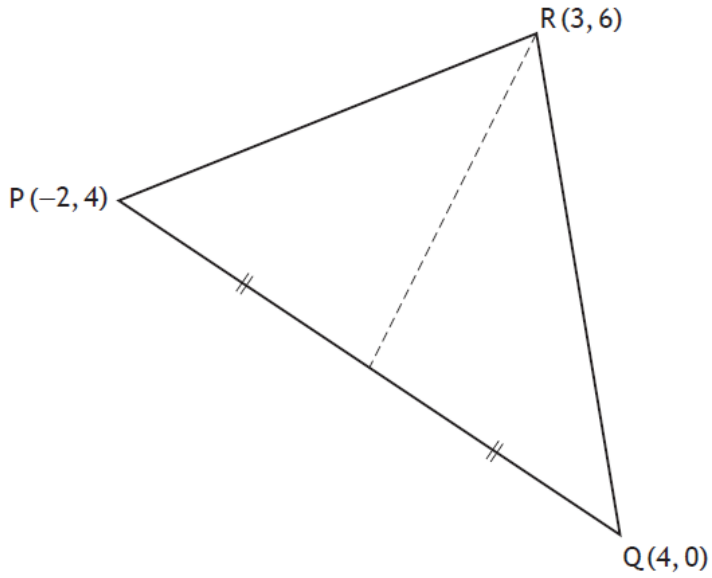
Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Attempt ALL questions

Total marks — 60

1. PQR is a triangle with vertices P(−2, 4), Q(4, 0) and R(3, 6).



Find the equation of the median through R. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/bkmaA5LLQLA

Video Lesson: 1·8 Silver Outcome 2

2. A function $g(x)$ is defined on $\mathbb{R}$, the set of real numbers, by

$$g(x) = \frac{1}{5}x - 4.$$

Find the inverse function, $g^{-1}(x)$. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/La0ZqajCMXA

Video Lesson: 3·3 Outcome 1

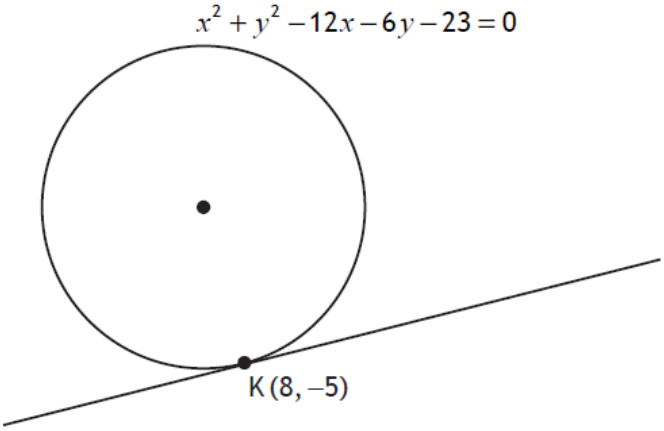
3. Given $h(x) = 3\cos 2x$, find the value of $h'\left(\frac{\pi}{6}\right)$. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/9vwhWt4eJho

Video Lesson: 13·1 Silver Outcome 2

4. The point K (8, –5) lies on the circle with equation $x^2 + y^2 - 12x - 6y - 23 = 0$.



Find the equation of the tangent to the circle at K. 4

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/prE9CvcsQ2w

Video Lesson: 11·2 Silver Outcome 2

5. A (–3, 4, –7), B (5, t, 5) and C (7, 9, 8) are collinear.

(a) State the ratio in which B divides AC. 1

(b) State the value of t. 1

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/h9me_sigK5w

Video Lesson: 12·2 Silver Outcome 2

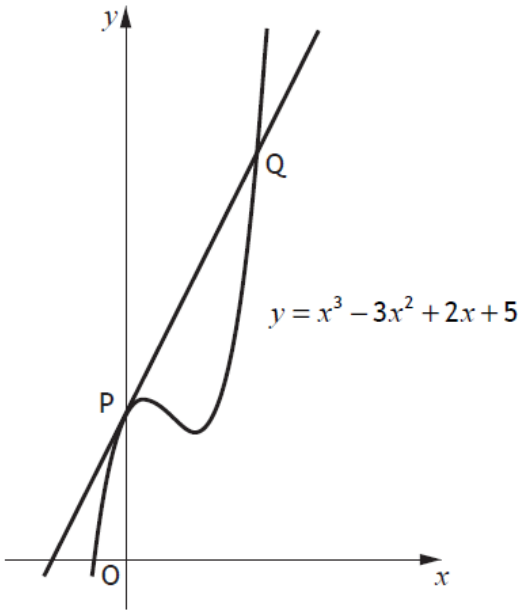
6. Find the value of $\log_5 250 - \frac{1}{3}\log_5 8$. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/2BngDfZ_jcM

Video Lesson: 14·1 Gold Outcome 3

7. The curve with equation $y = x^3 - 3x^2 + 2x + 5$ is shown on the diagram.



(a) Write down the coordinates of P, the point where the curve crosses the y-axis . 1

(b) Determine the equation of the tangent to the curve at P. 3

(c) Find the coordinates of Q, the point where this tangent meets the curve again. 4

Scan the QR code or click on it to view the worked solutions;

https://youtu.be/wgE0kP_jgrE

Video Lessons: 6·3 Silver Outcome 2, 7·3 Silver Outcome 2

8. A line has equation $y - \sqrt{3}x + 5 = 0$.

Determine the angle this line makes with the positive direction of the x-axis. 2

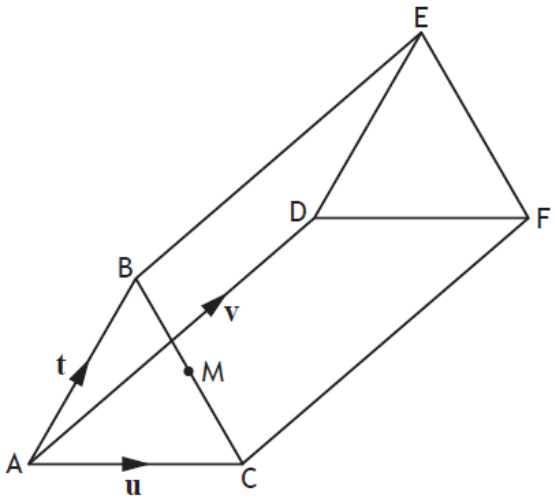
Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/RGkrIIKunO0

Video Lesson: 1·3 Gold Outcome 3

9. The diagram shows a triangular prism ABC,DEF.

$\overrightarrow{AB} = \mathbf{t}$, $\overrightarrow{AC} = \mathbf{u}$ and $\overrightarrow{AD} = \mathbf{v}$.



(a) Express $\overrightarrow{BC}$ in terms of $\mathbf{u}$ and $\mathbf{t}$. 1

M is the midpoint of BC.

(b) Express $\overrightarrow{MD}$ in terms of $\mathbf{t}$, $\mathbf{u}$ and $\mathbf{v}$. 2

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/xyXpB6Vv91w

Video Lesson: 12·1 Gold Outcome 3

10. Given that

- $\frac{dy}{dx} = 6x^2 - 3x + 4$, and
- $y = 14$ when $x = 2$,

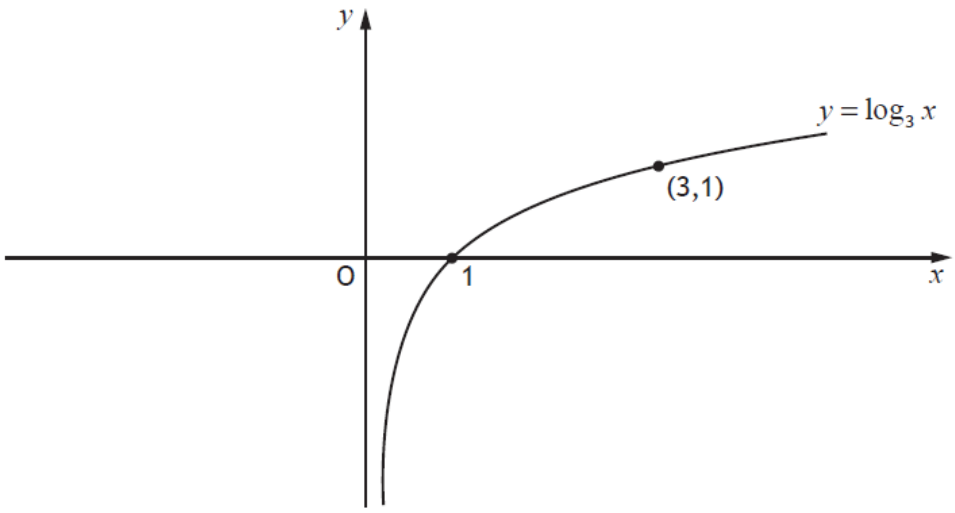
express y in terms of x. 4

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/xauQk607Ft4

Video Lesson: 9·3 Outcome 1

11. The diagram shows the curve with equation $y = \log_3 x$.



(a) On the diagram in your answer booklet, sketch the curve with equation $y = 1 - \log_3 x$. 2

(b) Determine the exact value of the x-coordinate of the point of intersection of the two curves. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/QBbyaQc_sCA

Video Lessons: 4·1 Bronze Outcome 1, 14·2 Bronze Outcome 1

12. Vectors $\mathbf{a}$ and $\mathbf{b}$ are such that $\mathbf{a} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + \mathbf{j} + p\mathbf{k}$.

(a) Express $2\mathbf{a} + \mathbf{b}$ in component form. 1

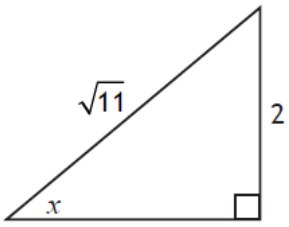
(b) Hence find the values of p for which $|2\mathbf{a} + \mathbf{b}| = 7$. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/GczI1A4dACg

Video Lessons: 12·1 Bronze Outcome 1

13. The right-angled triangle in the diagram is such that $\sin x = \frac{2}{\sqrt{11}}$ and $0 < x < \frac{\pi}{4}$.



(a) Find the exact value of:

(i) $\sin 2x$ 3

(ii) $\cos 2x$. 1

(b) By expressing $\sin 3x$ as $\sin(2x + x)$, find the exact value of $\sin 3x$. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/OdnTBolwwnM

Video Lessons: 10·1 Bronze, Silver and Gold Outcomes 1-3

14. Evaluate $\int_{-4}^{9} \frac{1}{\sqrt[3]{(2x+9)^2}}\,dx$. 5

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/bCqGHqEtok8

Video Lesson 1: 13·2 Gold Outcome 3

15. A cubic function, f, is defined on the set of real numbers.

- $(x+4)$ is a factor of $f(x)$
- $x=2$ is a repeated root of $f(x)$
- $f'(-2)=0$
- $f'(x)>0$ where the graph with equation $y=f(x)$ crosses the y-axis

Sketch a possible graph of $y=f(x)$ on the diagram in your answer booklet. 4

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/DPm6wdlBNX4

Video Lesson: 6·5 Gold Outcome 3

[END OF QUESTION PAPER]