Higher Mathematics

2019 Paper 2

Time allowed = 1 hr 45 mins Marks available = 80

For each question, you can scan the QR codes if using a paper copy or click on the links viewing this document electronically. This will allow you to view the worked solutions for each question. You can also either scan this QR Code or click on the link below to view this paper's marking scheme;

https://www.sqa.org.uk/pastpapers/papers/instructions/2019/mi_NH_Mathematics_all_2019.pdf

Remember to record your percentage for this paper in your analysis grid (your score ÷ 80 × 100).

FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product

$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

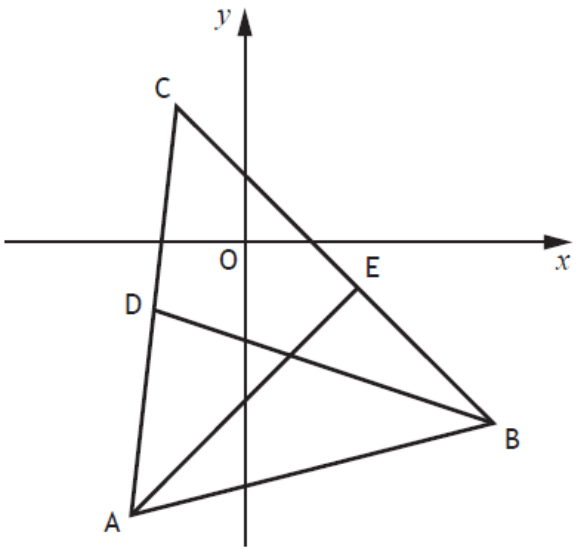
Table of standard integrals

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Attempt ALL questions

Total marks — 80

1. Triangle ABC has vertices A(−5,−12), B(11,−8) and C(−3,6).



(a) Find the equation of the median BD. 3

(b) Find the equation of the altitude AE. 3

(c) Find the coordinates of the point of intersection of BD and AE. 2

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/4M3LJtq94zY

Video Lessons:

1·8 Bronze Outcome 1

1·8 Silver Outcome 2

1·9 Bronze Outcome 1

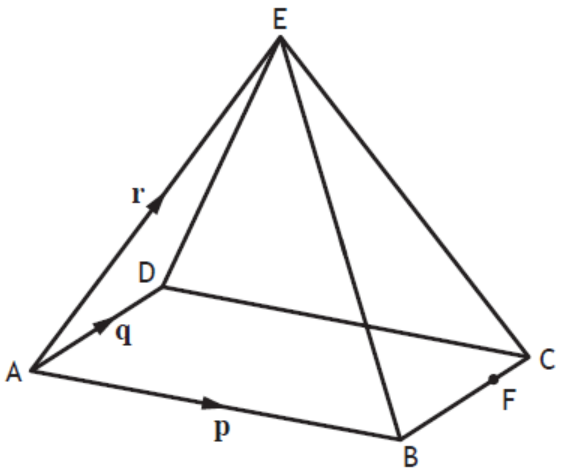
2. Find $\int\left(6\sqrt{x}-4x^{-3}+5\right)dx$. 4

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/hKfcdcMNfqM

Video Lesson: 9·1 Silver Outcome 2

3. E,ABCD is a rectangular based pyramid.
$\overrightarrow{AB}=\mathbf{p}$, $\overrightarrow{AD}=\mathbf{q}$ and $\overrightarrow{AE}=\mathbf{r}$.



(a) Express $\overrightarrow{BE}$ in terms of $\mathbf{p}$ and $\mathbf{r}$. 1

Point F divides BC in the ratio 3:1.

(b) Express vector $\overrightarrow{EF}$ in terms of $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$. 2

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/Y4jjGL_vISI

Video Lesson: 12·1 Gold Outcome 3

4. In a forest, the population of a species of mouse is falling by 2·7% each year.

   To increase the population scientists plan to release 30 mice into the forest at the end of March each year.

   (a) u_n is the estimated population of mice at the start of April, n years after the population was first estimated.

   It is known that u_n and u_{n+1} satisfy the recurrence relation $u_{n+1} = au_n + b$.

   State the values of a and b. 1

   The scientists continue to release this species of mouse each year.

   (b) (i) Explain why the estimated population of mice will stabilise in the long term. 1

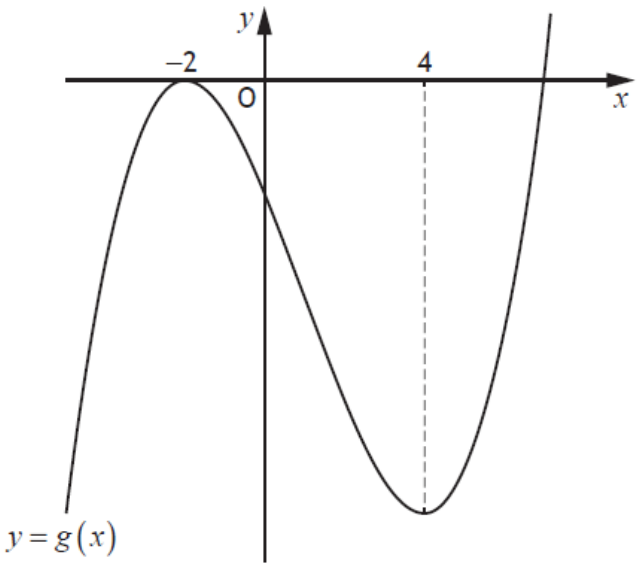
   (ii) Calculate the long term population to the nearest hundred. 2

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/IYD37iVKEwI

Video Lesson: 2·4 Outcome 1

5. The diagram below shows the graph of a cubic function $y = g(x)$, with stationary points at $x = -2$ and $x = 4$.



On the diagram in your answer booklet, sketch the graph of $y = g'(x)$. 2

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/BmeZ_wofua4

Video Lesson: 6·4 Gold Outcome 3

6. (a) Express $2\cos x° - 3\sin x°$ in the form $k\cos(x+a)°$ where $k>0$ and $0 \le a < 360$. 4

(b) Hence solve $2\cos x° - 3\sin x° = 3$ for $0 \le x < 360$. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/AkJTtpGN4WY

Video Lesson: 15·1 Bronze Outcome 1 and 15·2 Bronze Outcome 1

7. (a) Express $-6x^2 + 24x - 25$ in the form $p(x+q)^2 + r$. 3

(b) Given that $f(x) = -2x^3 + 12x^2 - 25x + 9$,
show that $f(x)$ is strictly decreasing for all $x \in \mathbb{R}$. 3

Scan the QR code or click on it to view the worked solutions;

https://youtu.be/N9bTGzGAxsI

Video Lesson: 8·2 Silver Outcome 2

8. A function, f, is given by $f(x) = \sqrt[3]{x} + 8$.

The domain of f is $1 \leq x \leq 1000,\ x \in \mathbb{R}$.

The inverse function, f^{-1}, exists.

(a) Find $f^{-1}(x)$. 3

(b) State the domain of f^{-1}. 1

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/BdFOpEk8Geo

Video Lesson: 3·3 Outcome 1

9. Electricity on a spacecraft can be produced by a type of nuclear generator.

The electrical power produced by this generator can be modelled by

$$P_t = 120e^{-0.0079t}$$

where P_t is the electrical power produced, in watts, after t years.

(a) Determine the electrical power initially produced by the generator. 1

(b) Calculate how long it takes for the electrical power produced by the generator to reduce by 15%. 4

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/jA3tHRPRFAQ

Video Lesson: 14·3 Gold Outcome 3

10. (a) Show that $(x+3)$ is a factor of $3x^4+10x^3+x^2-8x-6$. 2

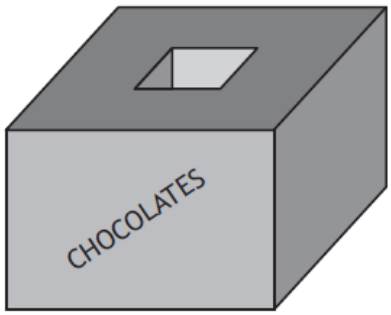
(b) Hence, or otherwise, factorise $3x^4+10x^3+x^2-8x-6$ fully. 5

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/eUIt-k8jTVc

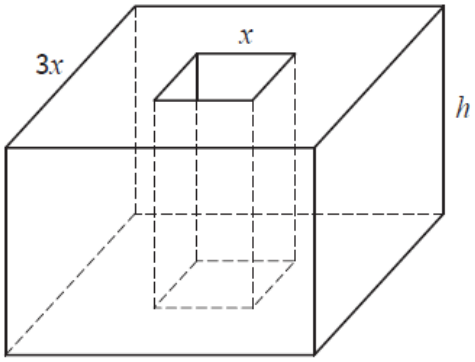
Video Lesson: 7·1 Gold Outcome 3

11. A manufacturer of chocolates is launching a new product in novelty shaped cardboard boxes.



The box is a cuboid with a cuboid shaped tunnel through it.

- The height of the box is h centimetres
- The top of the box is a square of side $3x$ centimetres
- The end of the tunnel is a square of side x centimetres
- The volume of the box is $2000\,\text{cm}^3$



(a) Show that the total surface area, A cm^2, of the box is given by

$$A = 16x^2 + \frac{4000}{x}.$$ 3

(b) To minimise the cost of production, the surface area, A, of the box should be as small as possible.

Find the minimum value of A. 6

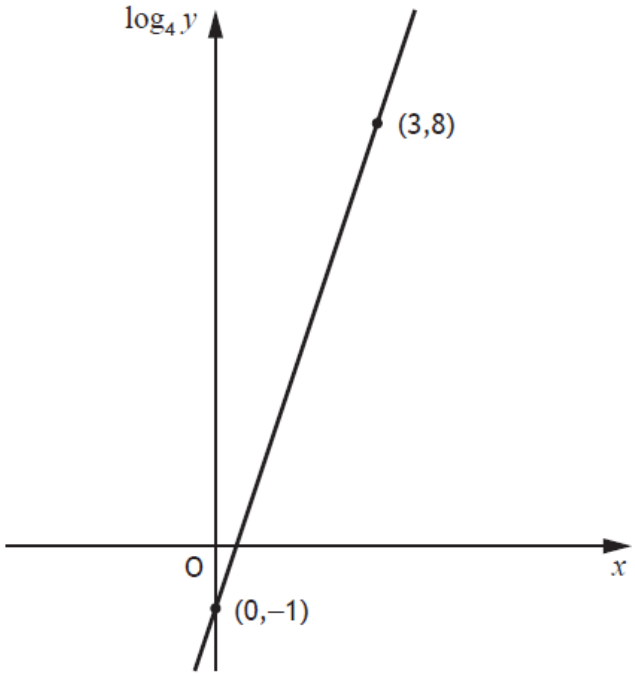
Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/aN2HIqxEftU

Video Lesson: 6·8 Outcome 1

12. Two variables, x and y, are connected by the equation $y = ab^x$.

The graph of $\log_4 y$ against x is a straight line as shown.



Find the values of a and b. 5

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/DiyJxw5NwQg

Video Lesson: 14·4 Silver Outcome 2

13. For a function, f, defined on the set of real numbers, $\mathbb{R}$, it is known that

- the rate of change of f with respect to x is given by $3x^2 - 16x + 11$
- the graph with equation $y = f(x)$ crosses the x-axis at (7,0).

Express $f(x)$ in terms of x. 5

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/2gc4pP4TGlg

Video Lesson: 9·3 Outcome 1

14. The vectors $\mathbf{u}$ and $\mathbf{v}$ are such that

- $|\mathbf{u}| = 4$
- $|\mathbf{v}| = 5$
- $\mathbf{u}.(\mathbf{u}+\mathbf{v}) = 21$

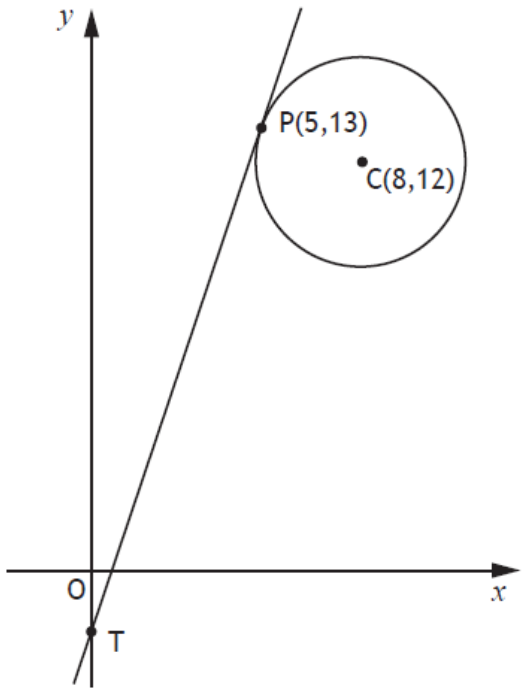
Determine the size of the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$. 4

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/8F-xrTtVzUo

Video Lessons: 12·3 Gold Outcome 3 and 12·4 Outcome 1

15. A circle has centre C(8,12).
The point P(5,13) lies on the circle as shown.



(a) Find the equation of the tangent at P. 3

The tangent from P meets the y-axis at the point T.

(b) (i) State the coordinates of T. 1

(ii) Find the equation of the circle that passes through the points C, P and T. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/qHehdkpnjMg

Video Lessons: 11·2 Silver Outcome 2 and 11·1 Gold Outcome 3

[END OF QUESTION PAPER]