# Higher Mathematics

# 2019 Paper 1

Time allowed = 1 hr 30 mins Marks available = 70

For each question, you can scan the QR codes if using a paper copy or click on the links viewing this document electronically. This will allow you to view the worked solutions for each question. You can also either scan this QR Code or click on the link below to view this paper's marking scheme;

https://www.sqa.org.uk/pastpapers/papers/instructions/2019/mi_NH_Mathematics_all_2019.pdf

Remember to record your percentage for this paper in your analysis grid (your score ÷ 70 × 100).

**FORMULAE LIST**

**Circle**

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a, b)$ and radius $r$.

**Scalar product**

$\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$, where $\theta$ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

**Trigonometric formulae**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

**Table of standard derivatives**

| $f(x)$ | $f'(x)$ |
|---|---|
| $\sin ax$ | $a\cos ax$ |
| $\cos ax$ | $-a\sin ax$ |

**Table of standard integrals**

| $f(x)$ | $\int f(x)dx$ |
|---|---|
| $\sin ax$ | $-\frac{1}{a}\cos ax + c$ |
| $\cos ax$ | $\frac{1}{a}\sin ax + c$ |

## Attempt ALL questions

## Total marks — 70

1. Find the $x$-coordinates of the stationary points on the curve with equation $y = \frac{1}{2}x^4 - 2x^3 + 6$. 4

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/lFwSr1edjNg

Video Lesson: 6·5 Bronze Outcome 1

2. The equation $x^2 + (k-5)x + 1 = 0$ has equal roots.
   Determine the possible values of $k$. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/75pgp6_XbUI

Video Lesson: 8·4 Gold Outcome 3

3. Circle $C_1$ has equation $x^2 + y^2 - 6x - 2y - 26 = 0$.
Circle $C_2$ has centre $(4,-2)$.
The radius of $C_2$ is equal to the radius of $C_1$.
Find the equation of circle $C_2$. 2

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/Pu4FREYy5Qc

Video Lesson: 11·1 Silver Outcome 2

4. A sequence is generated by the recurrence relation

$$u_{n+1} = mu_n + c,$$

where the first three terms of the sequence are 6, 9 and 11.

(a) Find the values of $m$ and $c$. 3

(b) Hence, calculate the fourth term of the sequence. 1

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/wzonvNkVCwI

Video Lesson: 2·3 Outcome 1

5. (a) Show that the points A(1,5,–3), B(4,–1,0) and C(8,–9,4) are collinear. 3

(b) State the ratio in which B divides AC. 1

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/IRXZUBZYz8Y

Video Lesson: 12·2 Silver Outcome 2

6. Given that $y = \frac{1}{(1-3x)^5}$, $x \neq \frac{1}{3}$, find $\frac{dy}{dx}$. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/fbFrDXMOO4M

Video Lesson: 13·1 Bronze Outcome 1

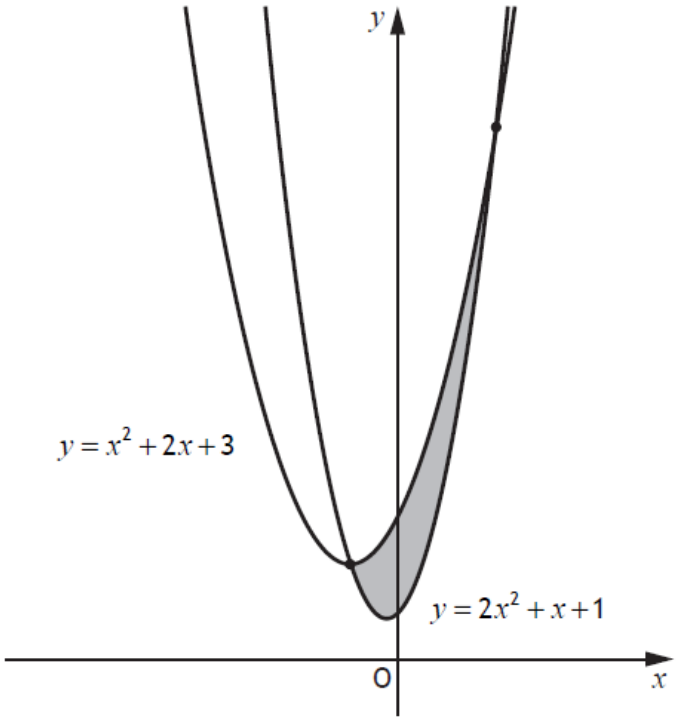
7. The line, $L$, makes an angle of 30° with the positive direction of the $x$-axis.
   Find the equation of the line perpendicular to $L$, passing through $(0,-4)$. 4

Scan the QR code or click on it to view the worked solutions;

https://youtu.be/IoY6z7foFYU

Video Lesson: 1·6 Gold Outcome 3

8. The graphs of $y = x^2 + 2x + 3$ and $y = 2x^2 + x + 1$ are shown below.




The graphs intersect at the points where $x = -1$ and $x = 2$.

(a) Express the shaded area, enclosed between the curves, as an integral. 1

(b) Evaluate the shaded area. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/TJHgjYN6DI8

Video Lesson: 9·4 Gold Outcome 3

9. Vectors $\mathbf{u}$ and $\mathbf{v}$ have components $\begin{pmatrix} p \\ -2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 2p+16 \\ -3 \\ 6 \end{pmatrix}$, $p \in \mathbb{R}$.

(a) (i) Find an expression for $\mathbf{u}.\mathbf{v}$. 1

(ii) Determine the values of $p$ for which $\mathbf{u}$ and $\mathbf{v}$ are perpendicular. 3

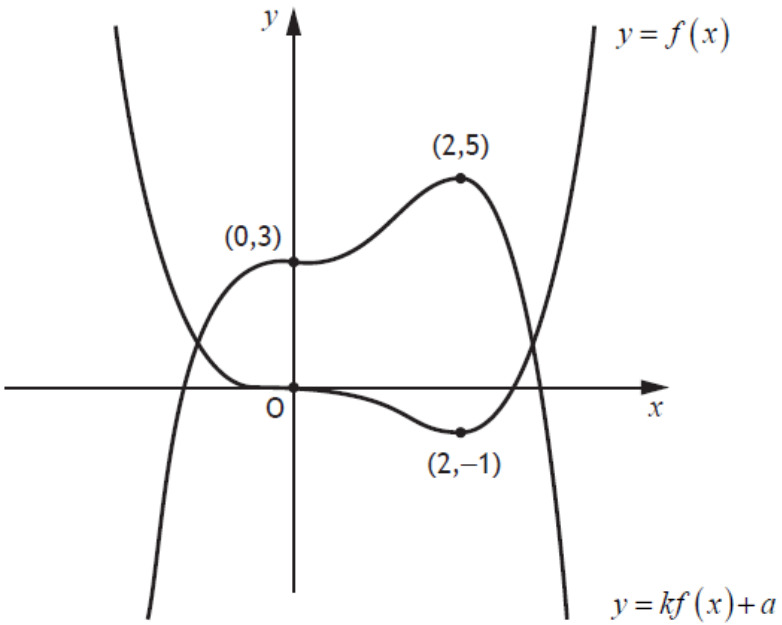
(b) Determine the value of $p$ for which $\mathbf{u}$ and $\mathbf{v}$ are parallel. 2

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/mfHFTlf0yz0

Video Lessons: 12·3 Bronze Outcome 1 and Silver Outcome 2

**10.** The diagram shows the graphs with equations $y = f(x)$ and $y = kf(x) + a$.




(a) State the value of $a$. 1

(b) Find the value of $k$. 1

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/x2VnVWoUBAU

Video Lesson: 4·1 Bronze Outcome 1

**11.** Evaluate $\int_0^{\frac{\pi}{9}} \cos\left(3x - \frac{\pi}{6}\right) dx$. 4

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/D-qpMSPLLTY

Video Lesson: 13·2 Gold Outcome 3

12. Functions $f$ and $g$ are defined by

- $f(x)=\frac{1}{\sqrt{x}}$, where $x>0$
- $g(x)=5-x$, where $x\in\mathbb{R}$.

(a) Determine an expression for $f(g(x))$. 2

(b) State the range of values of $x$ for which $f(g(x))$ is undefined. 1

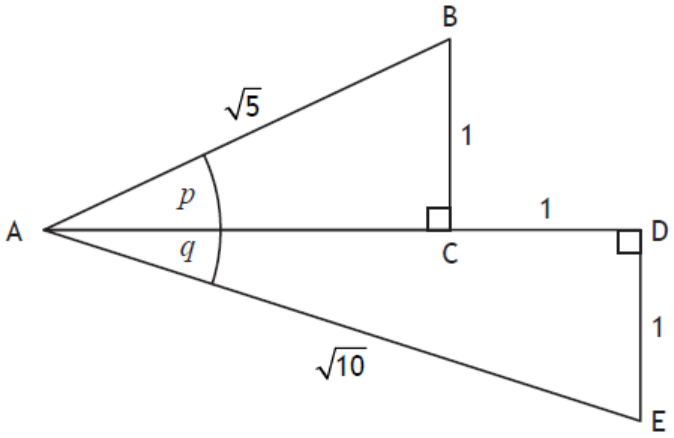
Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/wyWb89YjXi8

Video Lessons: 3·2 Silver Outcome 2 and 3·1 Silver Outcome 2

13. Triangles ABC and ADE are both right angled.

Angles $p$ and $q$ are as shown in the diagram.




(a) Determine the value of

(i) $\cos p$ 1

(ii) $\cos q$. 1

(b) Hence determine the value of $\sin(p+q)$. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/dWJ8-hLe9Ho

Video Lesson: 10·1 Gold Outcome 3

14. (a) Evaluate $\log_{10} 4 + 2\log_{10} 5$. 3

(b) Solve $\log_2(7x-2) - \log_2 3 = 5, \quad x \geq 1$. 3

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/h-bqRytZRVg

Video Lessons: 14·1 Gold Outcome 3 and 14·2 Silver Outcome 2

15. (a) Solve the equation $\sin 2x° + 6\cos x° = 0$ for $0 \leq x < 360$. 4

(b) Hence solve $\sin 4x° + 6\cos 2x° = 0$ for $0 \leq x < 360$. 1

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/rdnPSgmbHVs

Video Lesson: 10·2 Silver Outcome 2

16. The point P has coordinates $(4,k)$.

C is the centre of the circle with equation $(x-1)^2 + (y+2)^2 = 25$.

(a) Show that the distance between the points P and C is given by $\sqrt{k^2+4k+13}$. 2

(b) Hence, or otherwise, find the range of values of $k$ such that P lies outside the circle. 4

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/MPmLQW5PH4c

Video Lessons:

11·1 Bronze Outcome 1

1·4 Outcome 1

8·3 Silver Outcome 2

17. (a) Express $(\sin x - \cos x)^2$ in the form $p + q\sin rx$ where $p$, $q$ and $r$ are integers. 3

(b) Hence, find $\int (\sin x - \cos x)^2 \, dx$. 2

Scan the QR code or click on the link to view the worked solutions;

https://youtu.be/0PMKR3E9aEc

Video Lessons: 10·3 Gold Outcome 3 and 13·2 Gold Outcome 3

[END OF QUESTION PAPER]