X056/301

| NATIONAL QUALIFICATIONS 2000 | THURSDAY, 25 MAY 9.00 AM – 10.10 AM | MATHEMATICS HIGHER Paper 1 (Non-calculator) Units 1,2 and 3 |

Read Carefully

1 **Calculators may NOT be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

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FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $a.b = |a|\,|b| \cos \theta$, where θ is the angle between a and b

or $a.b = a_1b_1 + a_2b_2 + a_3b_3$ where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
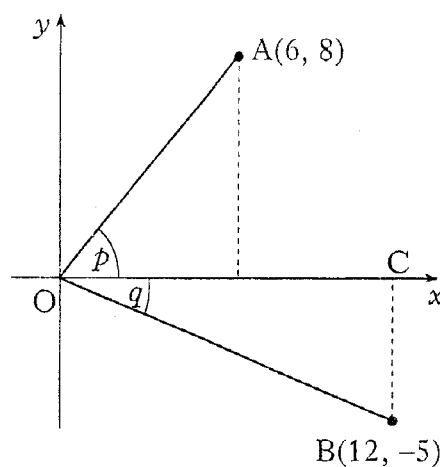
Table of standard derivatives and integrals:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

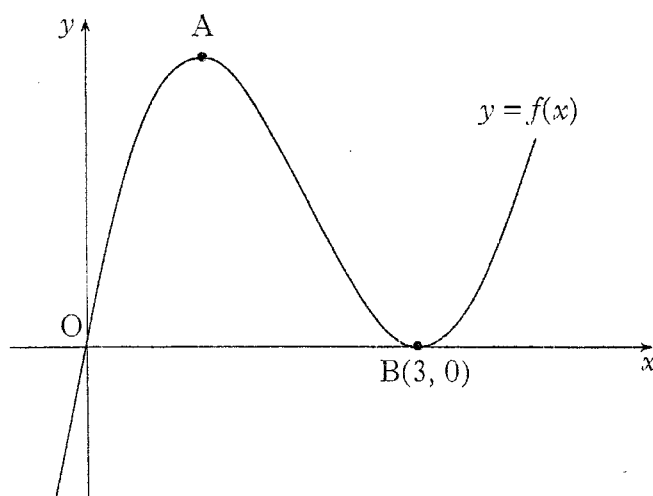| $f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\dfrac{1}{a}\cos ax + C$
$\cos ax$	$\dfrac{1}{a}\sin ax + C$

1. On the coordinate diagram shown, A is the point (6, 8) and B is the point (12, –5). Angle AOC = p and angle COB = q.

   Find the exact value of $\sin(p + q)$.   **4**

2. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below. The graph has a maximum at A and a minimum at B(3, 0).


   (a) Find the coordinates of the turning point at A.   **4**

   (b) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x + 2) + 4$.
       Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes.   **2**

   (c) Write down the range of values of k for which $g(x) = k$ has 3 real roots.   **1**
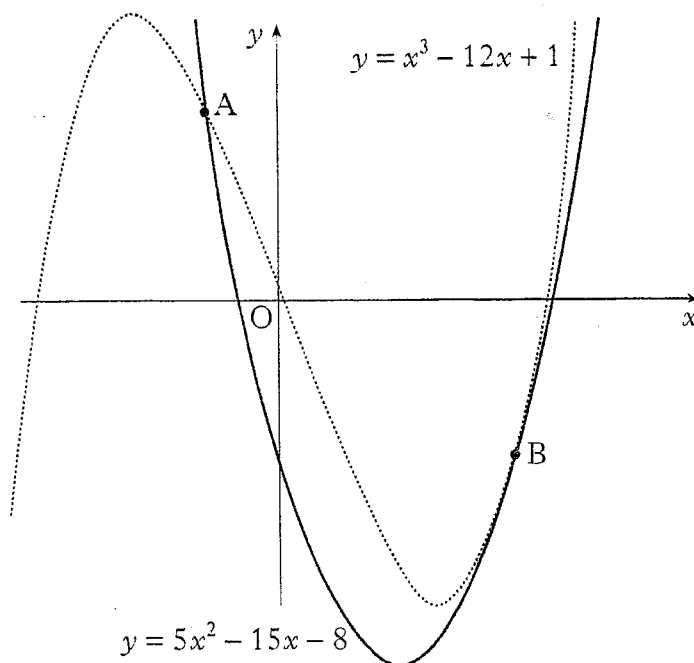
   [Turn over

Marks

3. Find the size of the angle $a°$ that the line joining the points A(0, −1) and B(3√3, 2) makes with the positive direction of the x-axis.
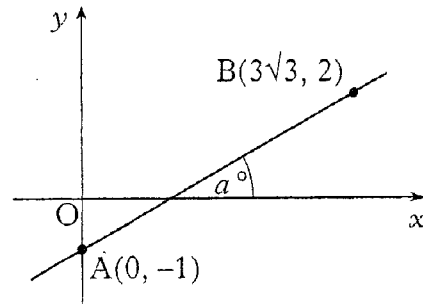
    3

4. The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$. The two curves intersect at A and touch at B, ie at B the curves have a common tangent.

    (a) (i) Find the x-coordinates of the points on the curves where the gradients are equal.

    4

    (ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i).

    1

    (b) The point A is (−1, 12) and B is (3, −8).

    Find the area enclosed between the two curves.

    5

Marks

5. Two sequences are generated by the recurrence relations $u_{n+1} = au_n + 10$ and $v_{n+1} = a^2 v_n + 16$.

   The two sequences approach the same limit as $n \to \infty$.

   Determine the value of a and evaluate the limit.   5

6. For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle?   5

7. VABCD is a pyramid with a rectangular base ABCD.
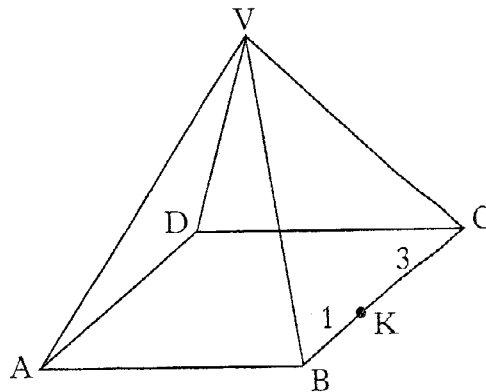
   Relative to some appropriate axes,

   $\overrightarrow{VA}$ represents $-7i - 13j - 11k$

   $\overrightarrow{AB}$ represents $6i + 6j - 6k$

   $\overrightarrow{AD}$ represents $8i - 4j + 4k$.

   K divides BC in the ratio 1:3.

   Find $\overrightarrow{VK}$ in component form.

3

8. The graph of $y = f(x)$ passes through the point $\left(\dfrac{\pi}{9}, 1\right)$.

   If $f'(x) = \sin(3x)$ express y in terms of x.   4

9. Evaluate $\log_5 2 + \log_5 50 - \log_5 4$.   3

10. Find the maximum value of $\cos x - \sin x$ and the value of x for which it occurs in the interval $0 \le x \le 2\pi$.   6

[END OF QUESTION PAPER]