X056/302

| NATIONAL QUALIFICATIONS 2000 | THURSDAY, 25 MAY 10.30 AM – 12.00 NOON | MATHEMATICS HIGHER Paper 2 Units 1,2 and 3 |

Read Carefully

1 **Calculators may be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $a.b = |a|\,|b|\cos\theta$, where θ is the angle between a and b

or $a.b = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
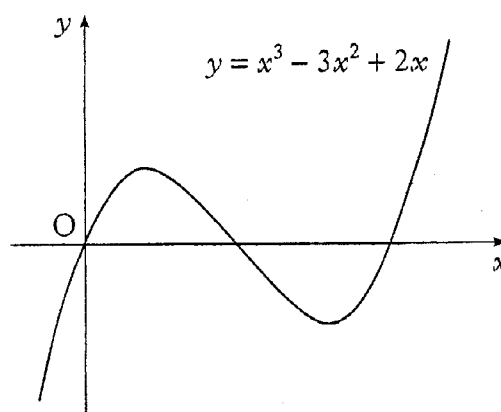$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

Table of standard derivatives and integrals:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\dfrac{1}{a}\cos ax + C$
$\cos ax$	$\dfrac{1}{a}\sin ax + C$

Marks

1. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.

   (a) Find the equation of the tangent to this curve at the point where $x = 1$.

   5

   (b) The tangent at the point (2, 0) has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again.
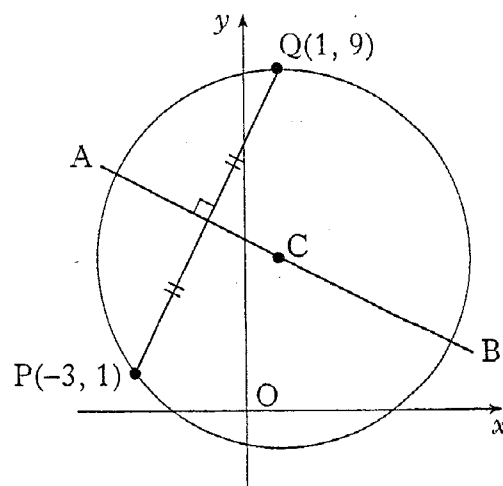
   5

   $y = x^3 - 3x^2 + 2x$

2. (a) Find the equation of AB, the perpendicular bisector of the line joining the points P(–3, 1) and Q(1, 9).

   4

   (b) C is the centre of a circle passing through P and Q. Given that QC is parallel to the y-axis, determine the equation of the circle.

   3

   (c) The tangents at P and Q intersect at T.

   Write down

   (i) the equation of the tangent at Q

   (ii) the coordinates of T.

   2

   Q(1, 9)

   A

   C

   B

   P(–3, 1)

3. $f(x) = 3 - x$ and $g(x) = \dfrac{3}{x}$, $x \neq 0$.

   (a) Find $p(x)$ where $p(x) = f(g(x))$.

   2

   (b) If $q(x) = \dfrac{3}{3 - x}$, $x \neq 3$, find $p(q(x))$ in its simplest form.
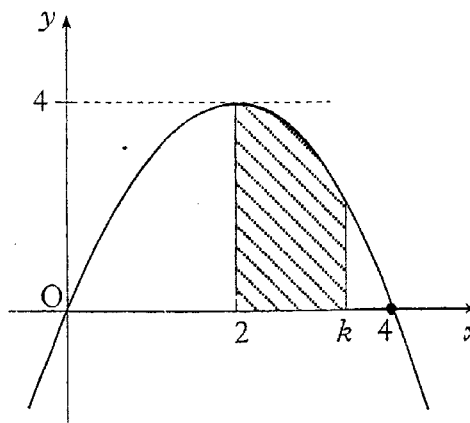
   3

Marks

4. The parabola shown crosses the x-axis at $(0, 0)$ and $(4, 0)$, and has a maximum at $(2, 4)$.

   The shaded area is bounded by the parabola, the x-axis and the lines $x = 2$ and $x = k$.

   (a) Find the equation of the parabola.   **2**

   (b) Hence show that the shaded area, A, is given by

   $$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}$$ **3**

5. Solve the equation $3 \cos 2x° + \cos x° = -1$ in the interval $0 \le x \le 360$.   **5**

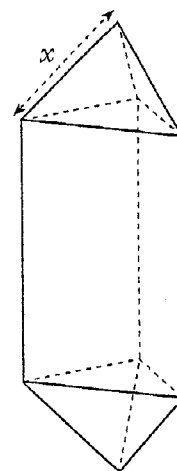6. A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end.

   The surface area, A, of the solid is given by

   $$A(x) = \frac{3\sqrt{3}}{2}\left(x^2 + \frac{16}{x}\right)$$

   where x is the length of each edge of the tetrahedron.

   Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid.   **6**
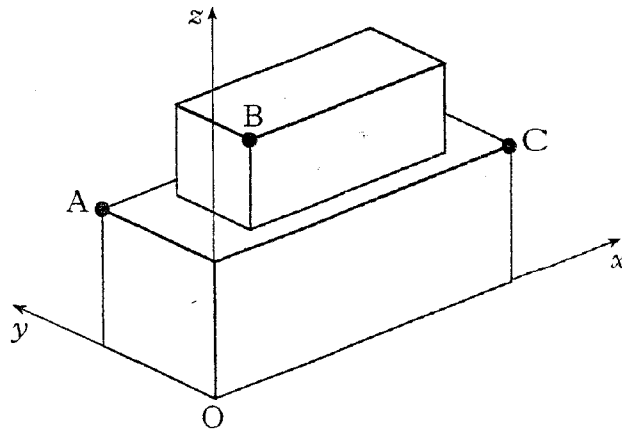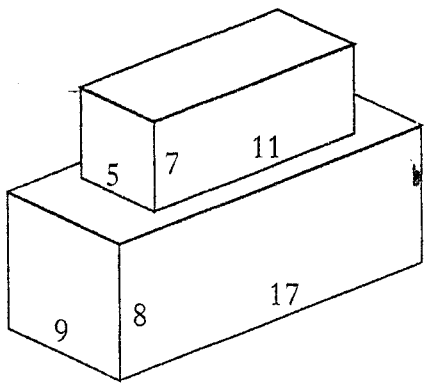
Marks

7. For what value of t are the vectors $u = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular?   2

8. Given that $f(x) = (5x - 4)^{\frac{1}{2}}$, evaluate $f'(4)$.   3

9. A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.

    Coordinate axes are taken as shown.

    (a) The point A has coordinates (0, 9, 8) and C has coordinates (17, 0, 8). Write down the coordinates of B.   1

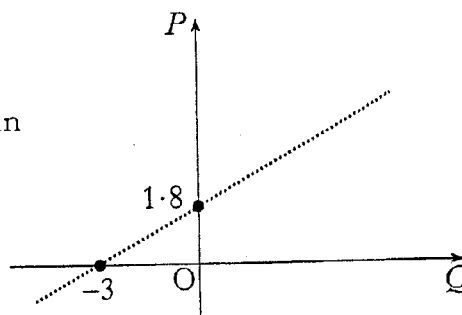   (b) Calculate the size of angle ABC.   6

[Turn over

Marks

10. Find $\int \dfrac{1}{(7-3x)^2}\, dx$.   2

11. The results of an experiment give
    rise to the graph shown.

    (a) Write down the equation of the line in
        terms of P and Q.   2

    It is given that $P = \log_e p$ and $Q = \log_e q$.

    (b) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the
        values of a and b.   4

[END OF QUESTION PAPER]