

X100/301

NATIONAL
QUALIFICATIONS
2004

FRIDAY, 21 MAY
9.00 AM – 10.10 AM

**MATHEMATICS
HIGHER**

Units 1, 2 and 3

Paper 1

(Non-calculator)

Read Carefully

- 1 Calculators may **NOT** be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b}

or $\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

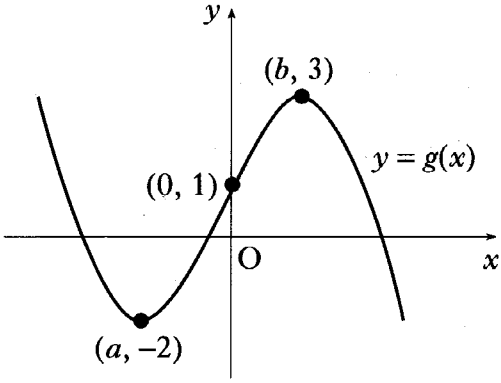
$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

ALL questions should be attempted.

Marks

1. The point A has coordinates (7, 4). The straight lines with equations $x + 3y + 1 = 0$ and $2x + 5y = 0$ intersect at B.
- (a) Find the gradient of AB. 3
- (b) Hence show that AB is perpendicular to only one of these two lines. 5
2. $f(x) = x^3 - x^2 - 5x - 3$.
- (a) (i) Show that $(x + 1)$ is a factor of $f(x)$.
- (ii) Hence or otherwise factorise $f(x)$ fully. 5
- (b) One of the turning points of the graph of $y = f(x)$ lies on the x -axis. Write down the coordinates of this turning point. 1
3. Find all the values of x in the interval $0 \leq x \leq 2\pi$ for which $\tan^2(x) = 3$. 4
4. The diagram shows the graph of $y = g(x)$.
- (a) Sketch the graph of $y = -g(x)$. 2
- (b) On the same diagram, sketch the graph of $y = 3 - g(x)$. 2
- 
5. A, B and C have coordinates $(-3, 4, 7)$, $(-1, 8, 3)$ and $(0, 10, 1)$ respectively.
- (a) Show that A, B and C are collinear. 3
- (b) Find the coordinates of D such that $\vec{AD} = 4\vec{AB}$. 2
6. Given that $y = 3\sin(x) + \cos(2x)$, find $\frac{dy}{dx}$. 3

[Turn over for Questions 7 to 11 on Page four]

5

7. Find $\int_0^2 \sqrt{4x+1} \, dx$.

8. (a) Write $x^2 - 10x + 27$ in the form $(x + b)^2 + c$.

2

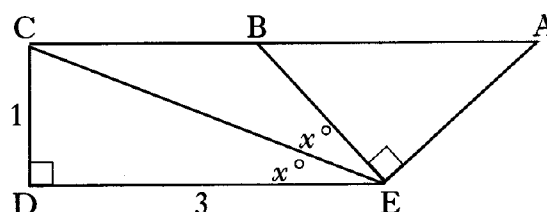
(b) Hence show that the function $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$ is always increasing.

4

9. Solve the equation $\log_2(x + 1) - 2\log_2(3) = 3$.

4

10. In the diagram
 angle $DEC = \text{angle } CEB = x^\circ$ and
 angle $CDE = \text{angle } BEA = 90^\circ$.
 $CD = 1$ unit; $DE = 3$ units.
 By writing angle DEA in terms
 of x° , find the exact value of
 $\cos(\angle DEA)$.



7

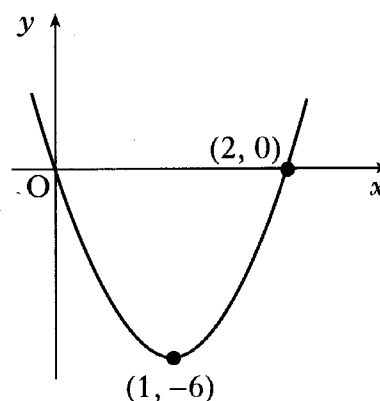
11. The diagram shows a parabola passing through the points $(0, 0)$, $(1, -6)$ and $(2, 0)$.

- (a) The equation of the parabola is of the form $y = ax(x - b)$.

Find the values of a and b .

- (b) This parabola is the graph of $y = f'(x)$.

Given that $f(1) = 4$, find the formula for $f(x)$.



3

5

[END OF QUESTION PAPER]