X100/301

NATIONAL QUALIFICATIONS 2002

MONDAY, 27 MAY
9.00 AM – 10.10 AM

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 1
(Non-calculator)

Read Carefully

1 **Calculators may NOT be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

LIB X100/301 6/27070

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos \theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted.

Marks

1. The point P(2, 3) lies on the circle $(x + 1)^2 + (y - 1)^2 = 13$. Find the equation of the tangent at P. **4**

2. The point Q divides the line joining P(–1, –1, 0) to R(5, 2, –3) in the ratio 2 : 1. Find the coordinates of Q. **3**

3. Functions f and g are defined on suitable domains by $f(x) = \sin(x^\circ)$ and $g(x) = 2x$.

   (*a*) Find expressions for:

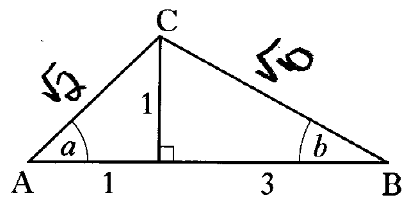
   (i) $f(g(x))$;

   (ii) $g(f(x))$. **2**

   (*b*) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$. **5**

4. Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of 45° with the positive direction of the x-axis. **4**

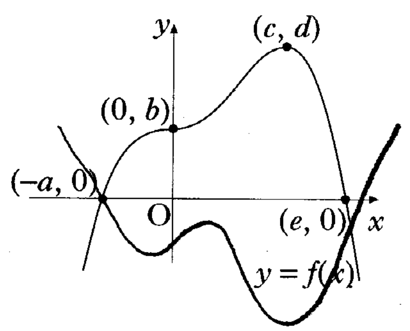
5. In triangle ABC, show that the exact value of $\sin(a + b)$ is $\frac{2}{\sqrt{5}}$. **4**



6. The graph of a function f intersects the x-axis at $(-a, 0)$ and $(e, 0)$ as shown.

   There is a point of inflexion at $(0, b)$ and a maximum turning point at (c, d).

   Sketch the graph of the derived function f'. **3**



[Turn over for Questions 7 to 11 on *Page four*

Marks

7. (*a*) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x - a)^2 + b$. **2**

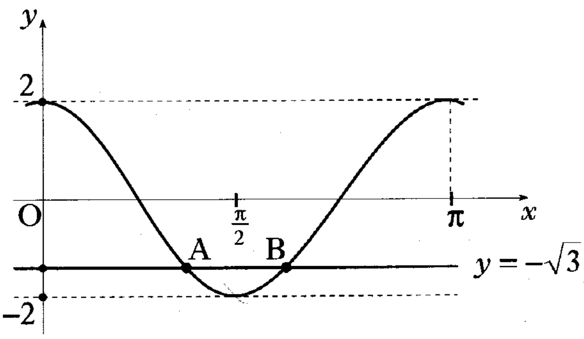
(*b*) On the same diagram sketch:

(i) the graph of $y = f(x)$;

(ii) the graph of $y = 10 - f(x)$. **4**

(*c*) Find the range of values of x for which $10 - f(x)$ is positive. **1**

8. The diagram shows the graph of a cosine function from 0 to π.



(*a*) State the equation of the graph. **1**

(*b*) The line with equation $y = -\sqrt{3}$ intersects this graph at points A and B.

Find the coordinates of B. **3**

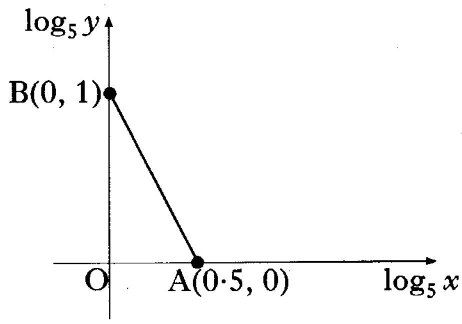
9. (*a*) Write $\sin(x) - \cos(x)$ in the form $k\sin(x - a)$ stating the values of k and a where $k > 0$ and $0 \leq a \leq 2\pi$. **4**

(*b*) Sketch the graph of $y = \sin(x) - \cos(x)$ for $0 \leq x \leq 2\pi$, showing clearly the graph's maximum and minimum values and where it cuts the x-axis and the y-axis. **3**

10. (*a*) Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}$, $x < 2$. **2**

(*b*) Hence write down $\int \frac{x^2}{(8 - x^3)^{\frac{1}{2}}} dx$. **1**

11. The graph illustrates the law $y = kx^n$. If the straight line passes through A(0·5, 0) and B(0, 1), find the values of k and n. **4**



[*END OF QUESTION PAPER*]